# MAT 122 2.0 Calculus

Dr. G.H.J. Lanel

Lecture 10

# Outline

1 L'hôpital's Rule: Indeterminate Forms

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## L'Hôpital's Rule for form 0/0

Suppose that $f$ and $g$ are differentiable functions on an open interval containing $x = a$, except possible at $x = a$, and that

$$\lim_{x \to a} f(x) = 0 \ \text{ and } \ \lim_{x \to a} g(x) = 0$$

If $\lim_{x \to a} \left[ \frac{f'(x)}{g'(x)} \right]$ exists, or if this limit is $+\infty$ or $-\infty$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Moreover this statement is also true in the case of limits as

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**E.g.** Find the limit

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

Using L'Hôpital's rule, and check the result by factoring.

**Sol:**

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \frac{2^2 - 4}{2 - 2} \quad 0/0 \; \textit{form}$$

Using L'Hôpital's rule

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{\frac{d}{dx}(x^2 - 4)}{\frac{d}{dx}(x - 2)}$$

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1. $\lim_{x\to 0} \frac{sin2x}{x}$
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**Sol:**

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$$\lim_{x \to 0} \frac{sin2x}{x} = \frac{sin0}{0} = \frac{0}{0}\ form$$

Applying L'Hôpital's rule

$$\lim_{x \to 0} \frac{sin2x}{x} = \lim_{x \to 0} \frac{\frac{d}{dx}(sin2x)}{\frac{d}{dx}(x)}$$

$$= \lim_{x \to 0} \frac{2cos2x}{1}$$

$$= 2cos(0) = 2$$

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**(2)**

$$\lim_{x\to\pi/2}\frac{1-sinx}{cosx}=\frac{1-sin\frac{\pi}{2}}{cos\pi/2}=\frac{1-1}{0}=\frac{0}{0}\ form$$

Applying L'Hôpital's rule

$$\lim_{x\to\pi/2}\frac{1-sinx}{cosx}=\lim_{x\to\pi/2}\frac{\frac{d}{dx}(1-sinx)}{\frac{d}{dx}(cosx)}$$
$$=\lim_{x\to\pi/2}\frac{-cosx}{-sinx}$$
$$=\frac{cos\pi/2}{sin\pi/2}$$
$$=\frac{0}{1}=0$$

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$$\lim_{x \to 0} \frac{e^x - 1}{x^3} = \frac{e^0 - 1}{0} = \frac{1 - 1}{0} = \frac{0}{0} \textit{ form}$$

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$$\lim_{x \to 0} \frac{e^x - 1}{x^3} = \lim_{x \to 0} \frac{\frac{d}{dx}\left(e^x - 1\right)}{\frac{d}{dx}\left(x^3\right)}$$

$$= \lim_{x \to 0} \frac{e^x}{3x^2}$$

$$= +\infty$$

**(3)**

$$\lim_{x\to 0}\frac{e^x-1}{x^3}=\frac{e^0-1}{0}=\frac{1-1}{0}=\frac{0}{0}\ \textit{form}$$

Applying L'Hôpital's rule

$$\lim_{x\to 0}\frac{e^x-1}{x^3}=\lim_{x\to 0}\frac{\frac{d}{dx}\left(e^x-1\right)}{\frac{d}{dx}\left(x^3\right)}$$
$$=\lim_{x\to 0}\frac{e^x}{3x^2}$$
$$=+\infty$$

**(3)**

$$\lim_{x \to 0} \frac{e^x - 1}{x^3} = \frac{e^0 - 1}{0} = \frac{1-1}{0} = \frac{0}{0} \textit{ form}$$

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$$\lim_{x\to 0^-} \frac{tan x}{x^2} = \frac{tan 0}{0} = \frac{0}{0} \ form$$

Applying L'Hôpital's rule

$$\lim_{x\to 0^-} \frac{tan x}{x^2} = \lim_{x\to 0^-} \frac{\frac{d}{dx}(tan x)}{\frac{d}{dx}(x^2)}$$

$$= \lim_{x\to 0^-} \frac{sec^2 x}{2x}$$

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# Indeterminate form of type $\infty/\infty$

The Limit of a ratio, $\frac{f(x)}{g(x)}$ in which the numerator has limit $\infty$ and the denominator has the limit $\infty$ is called an indeterminate form of type $\infty/\infty$

**L'Hôpital's Rule for $\infty/\infty$**

Suppose $f$ and $g$ are differentiable functions on an open interval containing $x = a$, except possibly at, $x = a$ and that

$$\lim_{x \to a} f(x) = \infty \text{ and } \lim_{x \to a} g(x) = \infty$$

If $\lim_{x \to a} \left[\frac{f'(x)}{g'(x)}\right]$ exists, or if this limit is $+\infty$ or $-\infty$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Moreover this statement is also true in the case of limits as

$x \longrightarrow a^-, x \longrightarrow a^+, x \longrightarrow -\infty$ or as $x \longrightarrow +\infty$.

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**E.g.** In each part confirm that the limit is an indeterminate form of type $\infty/\infty$ and evaluate it using L'HÔPITAL's rule.

1. $\lim_{x\to+\infty} \frac{x}{e^x}$
2. $\lim_{x\to 0^+} \frac{ln(x)}{csc(x)}$

**Sol:**

**(1)**

$$\lim_{x\to+\infty} \frac{x}{e^x} = \frac{\infty}{e^\infty} = \infty/\infty \ form$$

Applying L'Hôpital's rule

$$\lim_{x\to+\infty} \frac{x}{e^x} = \lim_{x\to+\infty} \frac{\frac{d}{dx}(x)}{\frac{d}{dx}(e^x)}$$
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**(2)**

$$\lim_{x \to 0^+} \frac{ln(x)}{csc(x)} = \frac{ln(0)}{csc(0)} = \infty/\infty \ form$$

Applying L'Hôpital's rule

$$\lim_{x \to 0^+} \frac{ln(x)}{csc(x)} = \lim_{x \to 0^+} \frac{\frac{d}{dx}(ln(x))}{\frac{d}{dx}(csc(x))}$$
$$= \lim_{x \to 0^+} \frac{\frac{1}{x}}{-csc(x)cot(x)}$$
$$= \infty/\infty \ form$$

Any additional application of L'Hôpital's rule will yield powers of $\frac{1}{x}$ in the numerator and expressions involving $csc(x)$ and $cot(x)$ in the denominator.

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Rewriting last expression

$$\lim_{x \to 0^+} \left( -\frac{sinx}{x} tanx \right) = -\lim_{x \to 0^+} \frac{sinx}{x} \lim_{x \to 0^+} tanx$$
$$= -(1)(0) = 0$$

Thus,

$$\lim_{x \to 0^+} \frac{ln(x)}{csc(x)} = 0$$

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The limit of an expression that has one of the forms

$$\frac{f(x)}{g(x)}, f(x)\cdot g(x), f(x)^{g(x)}, f(x)-g(x), f(x)+g(x)$$

is called and indeterminate form if the limits $f(x)$ and $g(x)$ individually exert conflicting influences on the limit of the entire expression.

**Indeterminate form of type** $0\cdot\infty$

For example

$\lim_{x\to 0^+} xln(x) = 0\cdot\infty$ Indeterminate form

On the other hand

$\lim_{x\to+\infty}\sqrt{x}\left(1-x^2\right) = +\infty(-\infty) = -\infty$ Not an indeterminate form

The limit of an expression that has one of the forms

$$\frac{f(x)}{g(x)}, f(x)\cdot g(x), f(x)^{g(x)}, f(x)-g(x), f(x)+g(x)$$

is called and indeterminate form if the limits $f(x)$ and $g(x)$ individually exert conflicting influences on the limit of the entire expression.

**Indeterminate form of type $0\cdot\infty$**

For example

$\lim_{x\to 0^+} xln(x) = 0\cdot\infty$ Indeterminate form

On the other hand

$\lim_{x\to +\infty} \sqrt{x}\,(1-x^2) = +\infty(-\infty) = -\infty$ Not an indeterminate form

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Indeterminate form of type $0 \cdot \infty$ can sometimes be evaluated by rewriting the product as a ratio, and then applying L'Hôpital's rule for indeterminate form of type 0/0 or $\infty/\infty$.

**E.g.** Evaluate

1. $lim_{x \to 0^+} xln(x)$
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**Sol:**

**(1)**

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## Applying L'Hôpital's rule

$$\begin{aligned}
\lim_{x\to 0^+} xln(x) = \lim_{x\to 0^+} \frac{ln(x)}{\frac{1}{x}} &= \lim_{x\to 0^+} \frac{\frac{d}{dx}\left(ln(x)\right)}{\frac{d}{dx}\left(\frac{1}{x}\right)} \\
&= \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} \\
&= \lim_{x\to 0^+} \frac{-x^2}{x} \\
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$$\lim_{x\to 0^+} x ln(x) = \lim_{x\to 0^+} \frac{ln(x)}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{d}{dx}(ln(x))}{\frac{d}{dx}\left(\frac{1}{x}\right)}$$

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$$\lim_{x \to \pi/4} (1 - tanx)(sec2x) = 0 \cdot \infty$$

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$$\lim_{x \to \pi/4} (1 - tanx)(sec2x) = \lim_{x \to \pi/4} \frac{1 - tanx}{\frac{1}{sec2x}}$$

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A limit problem that leads to one of the expressions

1. $(+\infty) - (+\infty)$
2. $(-\infty) - (-\infty)$
3. $(+\infty) + (-\infty)$
4. $(-\infty) + (-\infty)$

is called an indeterminate form type $\infty - \infty$

The limit problems that lead to one of the expressions

1. $(+\infty) + (+\infty)$
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are not indeterminate, since two terms work together.

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1. $(+\infty) + (+\infty)$
2. $(+\infty) - (-\infty)$
3. $(-\infty) + (+\infty)$
4. $(-\infty) - (+\infty)$

are not indeterminate, since two terms work together.

## Indeterminate form of type $\infty - \infty$

A limit problem that leads to one of the expressions

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**E.g.** Evaluate

$$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{sinx}\right)$$

**Sol:**

$$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{sinx}\right)=\left(\frac{1}{0}-\frac{1}{sin0}\right)=\infty-\infty \ form$$

Rewriting

$$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{sinx}\right)=\lim_{x\to 0^+}\left(\frac{sinx-x}{xsinx}\right)=\left(\frac{sin0-0}{0sin0}\right)=0/0 \ form$$

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$$\lim_{x \to 0^+} \left( \frac{1}{x} - \frac{1}{sinx} \right)$$

**Sol:**

$$\lim_{x \to 0^+} \left( \frac{1}{x} - \frac{1}{sinx} \right) = \left( \frac{1}{0} - \frac{1}{sin0} \right) = \infty - \infty \; form$$

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$$\lim_{x \to 0^+} \left( \frac{1}{x} - \frac{1}{sinx} \right) = \lim_{x \to 0^+} \left( \frac{sinx - x}{xsinx} \right) = \left( \frac{sin0 - 0}{0sin0} \right) = 0/0 \; form$$

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Applying L'Hôpital's rule

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## Again Applying L'Hôpital's rule

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## Indeterminate forms of type $0^0, \infty^0, 1^\infty$

Limits of the form

$$\lim f(x)g^{(x)}$$

can give rise to indeterminate forms of the types $0^0, \infty^0$ *and* $1^\infty$

**E.g.**

$$\lim_{x\to 0^+}(1+x)^{\frac{1}{x}} \quad (1^\infty) \textit{ form}$$

pause It is indeterminate because the expressions $1+x$ and $\frac{1}{x}$ gives 1 and $\infty$ respectively. Two conflicting influences. Such inderminate form can be evaluated by first introducing a dependent variable

$$y = f(x)^{g(x)}$$

$$In(y) = In(f(x)^{g(x)})$$

$$= g(x).In(f(x))$$

The limit of ln(y) will be an indeterminate form of type $0 \cdot \infty$

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**E.g.**

$$\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e \quad Note: \; a^x = e^x ln(a)$$

**Sol:** Let $y = (1+x)^{\frac{1}{x}}$

$$ln(y) = ln(1+x)^{\frac{1}{x}} \Rightarrow ln(y) = \frac{1}{x} ln(1+x)$$

$$\lim_{x\to 0} lny = \lim_{x\to 0} \frac{ln(1+x)}{x} = \frac{ln(1+0)}{0} \quad (0/0 \; form)$$

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