Cycles in Graph

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Outline

Cycles

Cycle

A cycle in a graph is a simple closed path.

Acyclic

A graph G is *acyclic* if it contains no cycles.

Maple command : IsAcyclic(G)

Girth

The *girth* of a graph is the length of its shortest cycle.

Simple graphs of minimal girth can be constructed by including a triangle, but achieving large girth in a graph with many edges is a difficult and interesting problem.
Maple command : Girth(G)

Königsberg bridge problem

The Seven Bridges of Königsberg is a historically notable problem in mathematics. The city of Königsberg in Prussia (now Kaliningrad, Russia) was set on both sides of the Pregel River, and included two large islands which were connected to each other, or to the two mainland portions of the city, by seven bridges.

Problem: Beginning anywhere and ending anywhere, can a person walk through town crossing all seven bridges but not crossing any bridge twice?

Eulerian Graph

Eulerian Trail

An *Eulerian trail (or Eulerian path)* is a trail in a finite graph which visits every edge exactly once.

A connected undirected graph contains an Eulerian path if and only if it contains exactly two vertices with odd degree.

Eulerian Cycle

An *Eulerian circuit or Eulerian cycle* is an Eulerian trail which starts and ends on the same vertex.

The term **Eulerian graph** has two common meanings in graph theory. One meaning is a graph with an Eulerian circuit, and the other is a graph with every vertex of even degree.
Maple command : IsEulerian(G)

Hamiltonian Graph

Hamiltonian Path

A Hamiltonian path is a path in an undirected or directed graph that visits each vertex exactly once.

Hamiltonian Cycle

A Hamiltonian cycle (or Hamiltonian circuit) is a Hamiltonian path which starts and ends on the same vertex.

A **Hamiltonian graph** is a graph that has a Hamiltonian cycle.
Maple command : IsHamiltonian(G)