Graph Coloring

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February 10, 2020

Outline

1. Graph Coloring
   - Vertex coloring/ Edge coloring/ Face coloring
2. Chromatic Number
3. Chromatic Polynomial
4. Applications of graph coloring

Graph coloring

In graph theory, graph coloring is a special case of graph labeling; it is an assignment of labels traditionally called "colors" to elements of a graph subject to certain constraints.

Vertex Coloring

Coloring of the the vertices of a graph such that no two adjacent vertices are of the same color; this is called a vertex coloring.

Edge Coloring

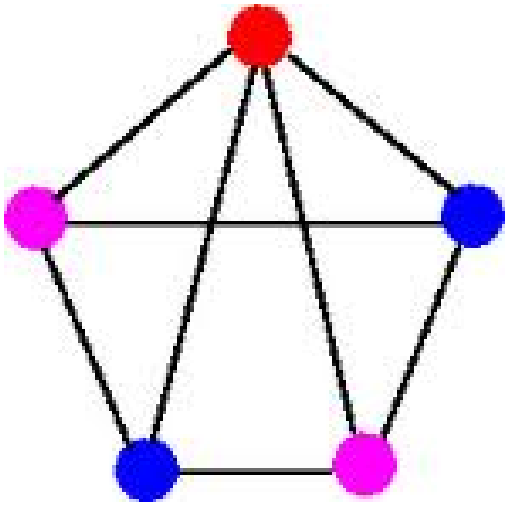
An edge coloring assigns a color to each edge so that no two adjacent edges are of the same color,

Face Coloring

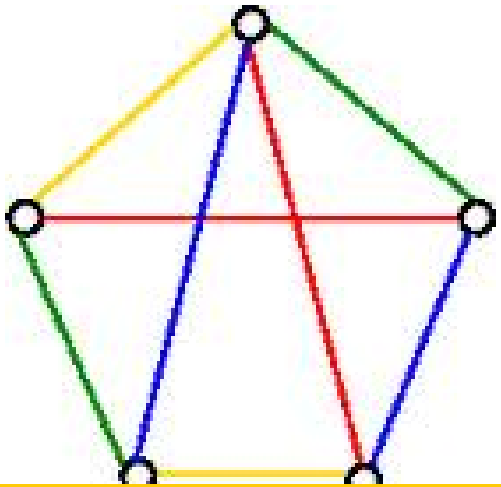
A face coloring of a planar graph assigns a color to each face or region so that no two faces that share a boundary have the same color.

Graph Coloring

Example : Vertex coloring

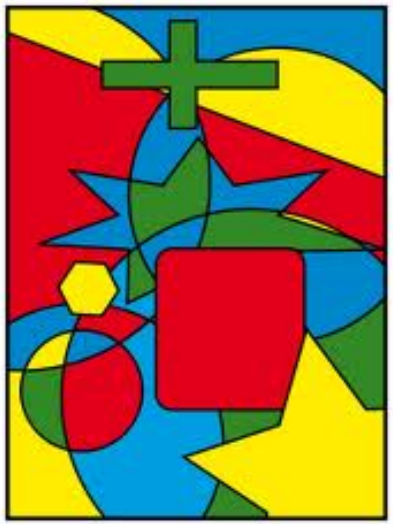


Example : Edge coloring



Graph Coloring

Example : Face coloring



Chromatic Number

A coloring a graph using at most k colors is called a (proper) k-coloring.

Chromatic Number

The smallest number of colors needed to color a graph G is called its chromatic number, and is often denoted by $\chi(G)$.

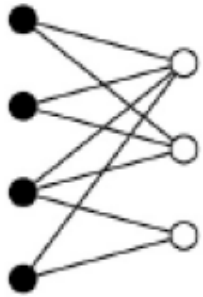
Maple Command : ChromaticNumber(G)

1. Chromatic number of a graph without having any edge is 1.
2. Chromatic number of a path(without having cycles) is 2.
3. The chromatic number of the n-cycle is given by

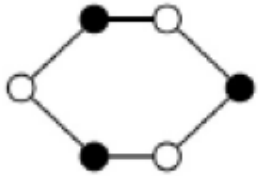
$$\chi(\text{n-cycle graph}) = \begin{cases} 2 & \text{if n is even} \\ 3 & \text{if n is odd} \end{cases}$$

Some examples of known chromatic numbers are;

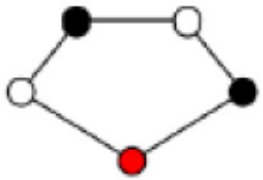
Bipartite graph
$\chi(G) = 2$



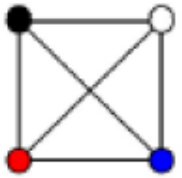
Even cycle
$\chi(G) = 2$



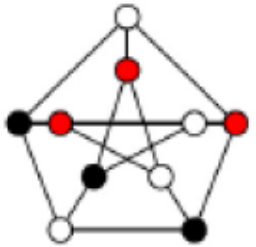
Odd cycle
$\chi(G) = 3$



$\chi(K_n) = n$



Petersen Graph
$\chi(G) = 3$



[K_n is a complete graph.]

Chromatic Polynomials

Chromatic Polynomials

The chromatic polynomial counts the number of ways a graph can be colored using no more than a given number of colors.

Maple Command : ChromaticPolynomial(G, 'x')

Edge Coloring

Edge Chromatic Number

Minimum number of colors need to do edge coloring of a graph.

Maple Command : EdgeChromaticNumber(G, 'col')

Applications of graph coloring

Problem 1: Sorting Fish

- A tropical fish hobbyist had six different types of fish: Alphas, Betas, Certas, Deltas, Epsalas, and Fetas, which shall henceforth be designated by A, B, C, D, E, and F, respectively.
- Because of predator-prey relationships, water conditions, and size, some fish can be kept in the same tank. The following table shows which fish cannot be together:

Type	A	B	C	D	E	F
Can not be with	B,C	A,C,D	A,B,D,E	B,C,F	C,F	D,E

- Question: What is the smallest number of tanks needed to keep all the fish?

Applications of graph coloring

Problem 2: Exam Scheduling

- Suppose that we want to schedule some final exams for MAT courses with following course numbers:

  1, 2, 3, 4, 5, 6, 7 and 8.

- Suppose also that there are no students in common taking the following pairs of courses: 1-2, 1-3, 1-4, 1-5, 1-6, 1-7, 1-8
  2-3, 2-5, 2-6, 2-7, 2-8
  3-8
  4-5, 4-6
  5-6

- How many exam slots are necessary to schedule exams?

Another application graph coloring

Problem 3 : Exam scheduling

- Suppose that in a particular quarter there are students taking each of the following combinations of courses:
  Math, English, Biology
  Math, English, Computer Science, Geography
  Biology, French, Geography
  Biology, Computer Science, French
  English, Computer Science, Math
  Computer Science, French
- What is the minimum number of examination periods required for the exams in the ten courses specified so that students taking any of the given combinations of courses have no conflicts?