MA1302 Engineering Mathematics I

Dr. G.H.J. Lanel

Lecture 1- Complex Numbers

Outline

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- In the late middle ages, mathematicians discovered that if one were willing to allow for a new number, one whose square was -1, quite a lot of mathematics got simpler!

  (They particularly noticed that they could solve quadratic and cubic equations!)

- This **imaginary** number was therefore very useful.
- Over time, the term **imaginary** has stuck, even though scientists and engineers now use complex numbers all the time.
- It is now common agreement to write i as an entity that satisfies

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- More analysis of electrical wiring and electrical signaling uses complex numbers.

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Complex Numbers

- A complex number z is a number that can be written in the form $z = a + bi$ where a and b are real numbers.
- This form is known as the standard form or Cartesian form of z.
- a is the real part of z, denoted by $Re(z)$, b is the imaginary part of z, denoted by $Im(z)$.
- i is known as the imaginary unit, and note that $i^2 := -1$
- The set of all complex numbers is denoted by $\mathbb{C}$.

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Exercise 1

Identify the real part and imaginary part of each complex number.

1. $5 - 3i$
2. $i\sqrt{2}$
3. $\dfrac{i + \pi}{5}$
4. $\dfrac{4 - i\pi}{5}$

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Equality of two complex numbers

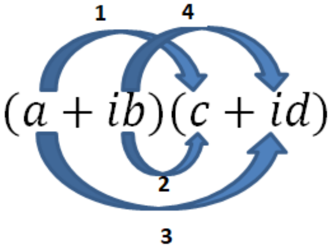
Suppose $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are two complex numbers.

Then, $z_1 = z_2$ if and only if $Re(z_1) = Re(z_2)$ and $Im(z_1) = Im(z_2)$.

Exercise

Let $z_1 = 2 + x - 3i$ and $z_2 = 5 + (1 - y)i$. If $z_1 = z_2$, find x and y.

Multiplication of complex numbers



Form the product terms of

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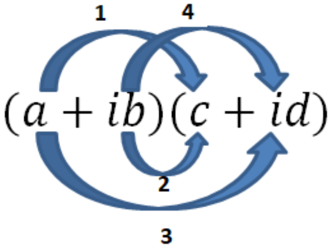
2. the two inner terms
3. the two outer terms
4. the two right-hand terms

Multiplication of complex numbers



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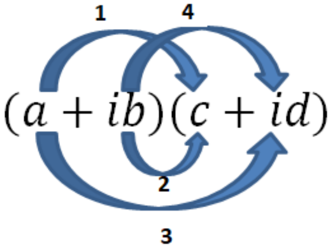

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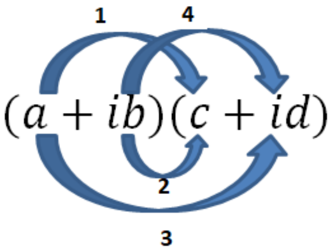

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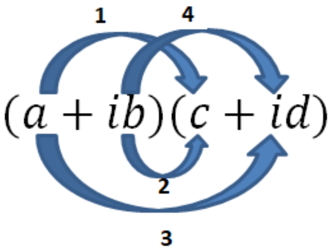

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Elementary operations on complex numbers

Exercise 2

Simplify the followings.

1. $(2+3i)+(i-4)$
2. $(3-i)-(1+5i)+(i-5)$
3. i^5
4. i^{-101}
5. $(2+3i)(i-4)$
6. $(3-i)(1+5i)(i-5)^2$
7. $\sqrt{-25}$
8. $(3-\sqrt{-16})(1+\sqrt{-9})$

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Conjugate of a complex number

If $z = a + bi$, then the conjugate of z, denoted by $\bar{z}$, is $a - ib$.

Exercise 3

Find the conjugate of each complex number.

1. $3 - 2i$
2. $i + 3$
3. π
4. $(3 + 2i)^2$

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Absolute value of a complex number

The absolute value (or modules) of a complex number $z = a + bi$, denoted by $|z|$, is given by $|z| = a^2 + b^2$:

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1. $|1 - i|$
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Properties of absolute value

Exercise 5

Let $z, w \in \mathbb{C}$: Prove the following. (Identify when the equality holds in each inequality.)

1. $|Re(z)| \leq |z|$
2. $|Im(z)| \leq |z|$
3. $|z| = |-z|$
4. $|z| = |\bar{z}|$
5. $z\bar{z} = |z|^2 = |\bar{z}|^2$
6. $|z| = 0$ if and only if $z = 0$
7. $|zw| = |z||w|$
8. $|\frac{z}{w}| = \frac{|z|}{|w|}$ where $w \neq 0$
9. $|z + w| \leq |z| + |w|$ (Triangular inequality)

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Division of complex numbers

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Divide as indicated and write your answer in standard form.

1. $\dfrac{3+2i}{4-3i}$
2. $\dfrac{1}{1-i}$
3. $\dfrac{a+bi}{c+di}$

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Exercise 7

Let $z, w \in \mathbb{C}$. Prove the following.

1. $Re(z) = \dfrac{z + \bar{z}}{2}$
2. $Im(z) = \dfrac{z - \bar{z}}{2i}$
3. $\bar{\bar{z}} = z$
4. $\overline{z + w} = \bar{z} + \bar{w}$
5. $\overline{zw} = \bar{z}\bar{w}$
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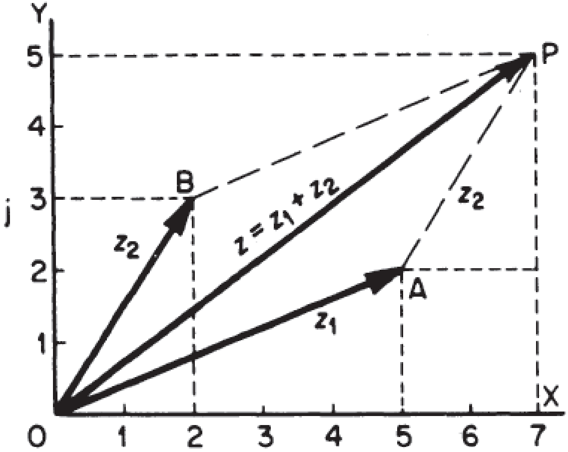
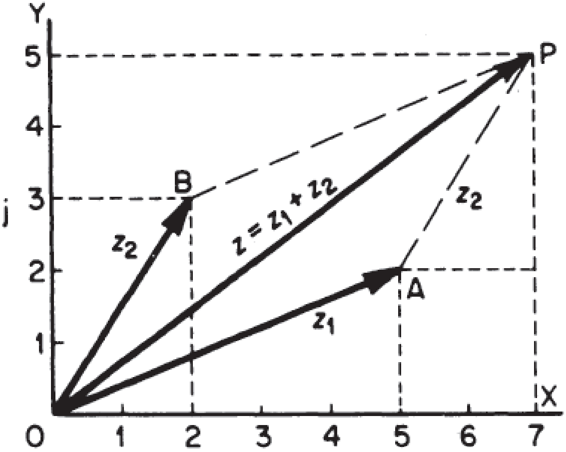


- Let us find the sum of z_1 and z_2 by Argand diagram.



- Let us find the sum of z_1 and z_2 by Argand diagram.

- If we are adding vectors, they must be drawn as a chain.
- We therefore draw at the end of z_1, a vector AP representing z_2 in magnitude and direction, i.e. AP = OB and is parallel to it.
- Therefore OAPB is a parallelogram.
- Thus the sum of z_1 and z_2 is given by the vector joining the starting point to the end of the last vector, i.e. OP.
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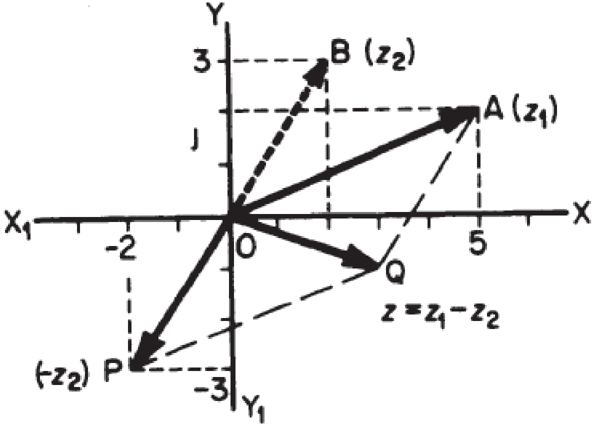
Y
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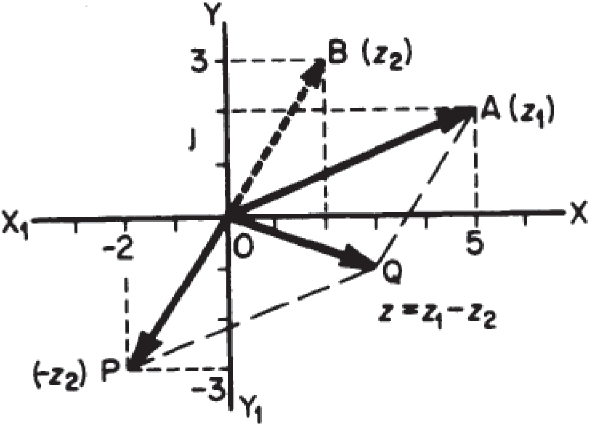
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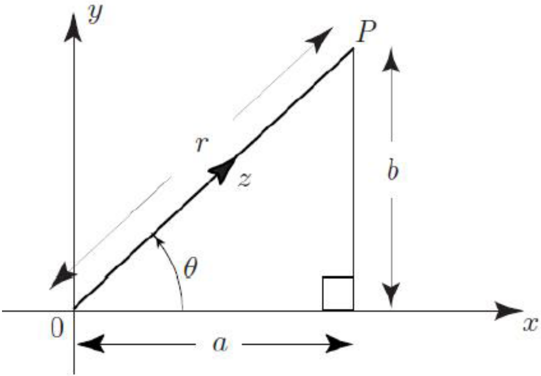
Outline

Let $z = a + bi \neq 0$.



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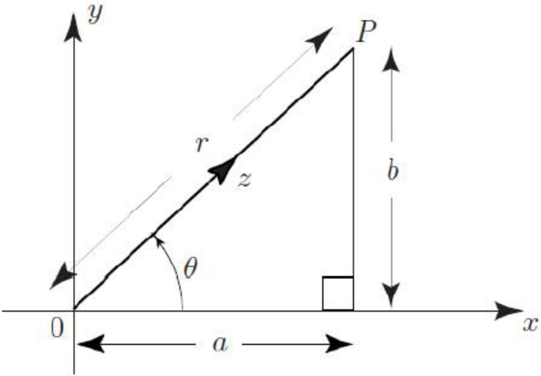

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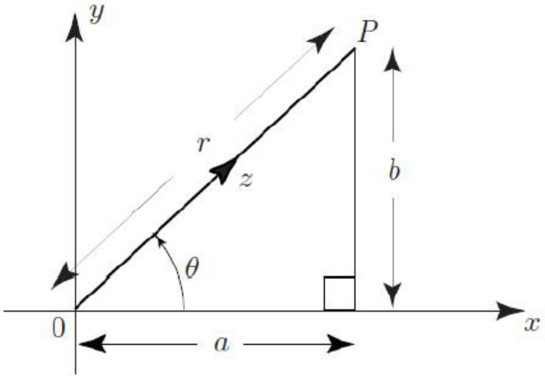

- The point P can be represented as (a, b) in rectangular coordinates and as (r, θ) in polar co-ordinates.

- Then z can be expressed using polar coordinates as $z = r(cos\theta + isin\theta)$ where $r = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}(\frac{b}{a})$.

- This is known as the polar form of $z, 0 < r$ is the modulus of z, and θ is the argument of z, denoted by $arg(z)$.

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Locate the given complex number on complex plane and give the polar form.

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- Let $z_1 = r_1(cos\theta_1 + isin\theta_1)$ and $z_2 = r_2(cos\theta_2 + isin\theta_2)$.
  Show that $z_1 z_2 = r_1 r_2[cos(\theta_1 + \theta_2) + isin(\theta_1 + \theta_2)]$.

- Let $z_1 = r_1(cos\theta_1 + isin\theta_1)$ and $0 \neq z_2 = r_2(cos\theta_2 + isin\theta_2)$.
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Outline

Euler's Formula

Euler's formula, named after Leonhard Euler, is a mathematical formula in complex analysis that establishes the fundamental relationship between the trigonometric functions and the complex exponential function. Euler's formula states that for any real number θ.

$$e^{i\theta} = cos\theta + isin\theta.$$

Let $z = r(cos\theta + isin\theta)$. Then $z = re^{i\theta}$ and this is known as the exponential form of z.

Exercise 11

Express the following in exponential form.

1. $1 + i$
2. $-\pi i$
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Let $n \in \mathbb{N}$. Then,

$$(cos\theta + isin\theta)^n = cos(n\theta) + isin(n\theta)$$

is called as the De Moivre's theorem.

Proof.

Prove De Moivre's theorem. □

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Exercise: Prove that, $cos3\theta = 4cos^3\theta - 3cos\theta$ and $sin3\theta = 3sin\theta - 4sin^3\theta$ by using De Moivre's theorem.

Outline

In general, a root is the value which makes polynomial or function as zero. Consider the polynomial,

$P(x) = a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n$ where $a_i \in \mathbb{C}, i = 1, \ldots, n$ and $n \in \mathbb{N}$. Then, $r_i, i = 1, \ldots, n$ is said to be a complex root of $p(x)$ when $r_i, \imath \in \mathbb{C}$ and $p(r_i) = 0$ for $i \in 1, 2, 3, \ldots, n$.

In the quadratic equation $ax^2 + bx + c = 0$ in which $b^2 - 4ac < 0$ has two complex roots. Therefore, whenever a complex number is a root of a polynomial with real coefficients, its complex conjugate is also a root of that polynomial.

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