MAT 122 2.0 Calculus

Dr. G.H.J. Lanel

Lecture 1

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Outline

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Outline

- 1 Introduction to Sequences
- Monotonic and Bounded sequence
- 3 Limit of a sequence
- 4 Convergent and Divergent sequences
- 5 Limit laws and limits of some important sequences



• we can obtain a **subsequence** by restricting the original sequence to a smaller index set. For example,

 $(a_2, a_4, a_6, ...)$ is a subsequence of $(a_1, a_2, a_3, a_4, ...)$

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The *n*th term

• A sequence is of the form *a*₁, *a*₂, *a*₃, *a*₄, ..., *a*_n, ...

The nth term of a sequence is denoted by a_n.

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 $\begin{array}{c} \mathbf{3} \quad \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots \end{array}$

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Answers

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, $\frac{1}{3}$, $\frac{1}{4}$, ..., $\Rightarrow a_n = \frac{1}{n}$
1, 2, 4, 8, 16, ..., $\Rightarrow a_n = 2^{n-1}$
 $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, ..., $\Rightarrow a_n = \frac{n}{n+1}$

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Outline

- Introduction to Sequences
- 2 Monotonic and Bounded sequence
- 3 Limit of a sequence
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Monotonic sequence

- A sequence is called **increasing**, if $a_n < a_{n+1}$ for all $n \ge 1$. That is $a_1 < a_2 < a_3$... for all n.
- It is called **decreasing**, if $a_n > a_{n+1}$ for all $n \ge 1$.
- It is called **monotonic** if it is either increasing or decreasing.

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- It is called monotonic if it is either increasing or decreasing.

Consider the following examples:

The sequence $\left\{rac{3}{n+5} ight\}$ is decreasing since

 $\frac{3}{n+5} > \frac{3}{(n+1)+5} = \frac{3}{n+6}$, for all $n \ge 1$.

(The right side is smaller because it has a larger denominator.)

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Show that the sequence
$$\left\{\frac{n}{n^2+1}\right\}$$
 is decreasing.

We must show that $a_{n+1} < a_n$, that is, $\frac{n+1}{(n+1)^2+1} < \frac{n}{n^2+1}$

$$\frac{n+1}{(n+1)^2+1} < \frac{n}{n^2+1} \Leftrightarrow (n+1)(n^2+1) < n[(n+1)^2+1] \\ \Leftrightarrow n^3 + n^2 + n + 1 < n^3 + 2n^2 + 2n \\ \Leftrightarrow 1 < n^2 + n$$

Since $n \ge 1$, $n^2 + 1 > 1$ we know that the inequality is true. Therefore, $a_{n+1} < a_n$ and so sequence is decreasing.

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• A sequence {*a_n*} is **bounded above** if there is a number *M* such that,

 $a_n \leq M$, for all $n \geq 1$.

• It is **bounded below** if there is a number *m* such that,

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 If it is bounded above and below, then {a_n} is a bounded sequence.

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Examples:

- The sequence $\{n\}$ is bounded below by 0 but not above.
- The sequence $\left\{\frac{n}{n+1}\right\}$ is bounded below and above by 0 and 1 respectively for all *n*.

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$\lim_{n\to\infty} a_n = L \text{ or } a_n \to L \text{ as } n \to \infty.$

i.e. if for every $\epsilon > 0$ there is a corresponding integer *N* such that $a_n - L < \epsilon$, whenever n > N.

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Convergent sequences have a finite limit.

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ..., \Rightarrow a_n = \frac{1}{n}$$
 and the limit of the sequence is 0

Divergent sequences do not have a finite limit.

 $3, 5, 7, 9, 11, 13, ..., \Rightarrow a_n = 2n + 1$ and the limit of the sequence is undefined.

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Outline

- Introduction to Sequences
- 2 Monotonic and Bounded sequence
- 3 Limit of a sequence
- 4 Convergent and Divergent sequences
- 5 Limit laws and limits of some important sequences

• $\lim_{n\to\infty}(a_n+b_n)=\lim_{n\to\infty}a_n+\lim_{n\to\infty}b_n$

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$$\lim_{n\to\infty}(a_n-b_n)=\lim_{n\to\infty}a_n-\lim_{n\to\infty}b_n$$

- $\lim_{n\to\infty} c * a_n = c * \lim_{n\to\infty} a_n$, for some real number c
- $\lim_{n\to\infty} a_n * \lim_{n\to\infty} b_n = \lim_{n\to\infty} a_n * b_n$
- If $\lim_{n\to\infty} b_n \neq 0$, then $\lim_{n\to\infty} a_n/b_n = \lim_{n\to\infty} a_n/\lim_{n\to\infty} b_n$

• If p > 0 and $a_n > 0$, then $\lim_{n \to \infty} a_n^p = (\lim_{n \to \infty} a_n)^p$

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• $\lim_{n\to\infty} (a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$

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- $\lim_{n\to\infty}(1+1/n)^n$
- $\bigoplus_{n\to\infty} n!/n^n$
- lim r^n , where *r* is a real number $n \to \infty$
- $\bigcup_{n\to\infty} n^{1/n}$

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- $\lim_{n\to\infty}(1+1/n)^n$
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