Functions

Dr. G.H.J. Lanel

Lecture 2

Outline

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Sets

- A **Set** is a well defined collection of objects.
  - Each object in a set is called **an element or member** of the set.
  - Use ,
    - *Upper case letters* to denote **a set**
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- A set that is not finite is called an *infinite set* .
- Two ways to describe sets.
  1. Listing method. Eg. $\{2, 1, 3, 0, 5\}$
  2. Rule method (Set builder method). Eg. $\{x \in \mathbb{Z} | 2 \leq x \leq 8\}$

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Exercise 1

1. Describe numbers between 0 and 100 using a set.

- Two sets A and B are equal if they have exactly the same elements and it writes $A = B$.
  ($A \neq B$ means "sets A and B do not have exactly the same elements.")

- If each element of a set A is also an element of set B, we say that A is a subset of B and write $A \subset B$.
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Sets Operations

- The *union* of sets A and B, denoted by $A \cup B$, is the set of all elements that are in A or B.

  $\mathrm{A}\cup\mathrm{B} = \{\mathrm{x}|\mathrm{x} \in A \text{ or } \mathrm{x} \in B\}$

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- If $A \cap B = \emptyset$ then say A and B are *disjoint*.
- The set of all elements under consideration is called the *universal set*, denoted by U.
- The complement of set A , denoted by A', is the set of all elements in U that are not in A.

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1. State true or false.
   1. $0 \in \{-1,1\}$
   2. $\emptyset \subset \{100\}$
   3. $\emptyset \in \{0\}$

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Sets of numbers

- The set of *natural numbers* : $\mathbb{N} = \{1, 2, 3, ...\}$
- The set of *integers*: $\mathbb{Z} = \{..., -2, -1, 0, 1, 2, 3, ...\}$
- The set of *rational numbers*: $\mathbb{Q}$ = $\{\frac{p}{q}$ where p,q are integers, q $\neq 0\}$
- The set of *irrational numbers*: $\mathbb{R} \setminus \mathbb{Q}$
- The set of *real numbers*: $\mathbb{R}$
- The set of complex numbers:$\mathbb{C}$ ={a+bi where a,b $\in \mathbb{R}$ and $i = \sqrt{-1}\}$

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Order of operations

PEMDAS:

1. Parenthesis
2. Exponents
3. Multiplication, Division (from left to right)
4. Addition, Subtraction (from left to right)

Exercise 3

What is $25^{\frac{1}{2}} - 6 \div 2(1+2) + 1$?

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Interval notation

- An *interval* is a set of real numbers with the property that any number that lies between two numbers in the set is also included in the set.
- *Closed interval* $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$
- *Open interval* $(a, b) = \{x \in \mathbb{R} : a < x < b\}$
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- $[a, \infty) = \{x \in \mathbb{R} : x \geq a\}$

Exercise 4

Describe the data set $\{2,3,4,5\} \cup (4, 9] \cap [-1, 10)$.

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Functions

- A quantity whose value can change is known as a *variable*.
- A quantity whose value cannot change is known as a *constant*.

Definition

Let A and B be two given nonempty sets.
A rule denoted by f is called a *function* if it corresponds each element (input) in A to a unique element (output) in B.

The set A is called the *domain* of f, set B is called the *codomain* of f, and the set of all the elements that return by f is called the *range* of f.

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A function may be expressed:

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Determine each relation given is a function.

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2. $f = \{(-1,-1),(0,1),(-1,1)\}$
3. $f = \{(-1,0),(0,1),(1,1)\}$
4. $x = y^2$
5. $y = x^2$
6. $x^2 + (y-2)^2 = 16$

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$$y = f(x) \text{ (read } y \text{ equals } f \text{ of } x\text{)}$$

- Mean for the given input value x the function returns y or $f(x)$ as the output.
- x is called the *independent variable* and y is called the *dependent variable*.

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2. Identify the domain and range of f.

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Domain of a function

The domain of a function is the set of all real numbers that return a real number as the output.

Exercise 7

Find the domain of each function. Give the answer in interval notation.

1. $f(x) = \sqrt{2x - 10}$
2. $g(x) = \dfrac{2x}{x + 5}$
3. $h(x) = e$
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Operations on functions

Let f be a function with domain D(f) and g be a function with domain D(g).

- **Sum** : $(f+g)(x) := f(x)+g(x)$ where $D(f+g) = D(f) \cap D(g)$
- **Difference** :$(f-g)(x) := f(x)-g(x)$ where $D(f-g) = D(f) \cap D(g)$
- **Product** :$(fg)(x) := f(x)g(x)$ where $D(fg) = D(f) \cap D(g)$
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- **Composition**: $(f \circ g)(x) := f[g(x)]$ where $D(f \circ g) = \{x \in D(g) : g(x) \in D(f)\}$

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- **Sum** : $(f+g)(x) := f(x)+g(x)$ where $D(f+g) = D(f) \cap D(g)$
- **Difference** :$(f-g)(x) := f(x)-g(x)$ where $D(f-g) = D(f) \cap D(g)$
- **Product** :$(fg)(x) := f(x)g(x)$ where $D(fg) = D(f) \cap D(g)$
- **Quotient**: $(f \div g)(x) := f(x) \div g(x)$ where $D(f \div g) = \{x \in$ D(f)$\cap$ D(g) : g(x) $\neq$0 $\}$
- **Composition**: $(f \circ g)(x) := f[g(x)]$ where $D(f \circ g) = \{x \in D(g) : g(x) \in D(f)\}$

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Exercise

Let $f(x) = \sqrt{x+1}$ and $g(x) = \frac{1}{x-1}$. Find indicated functions and give domain of each function.

1. $f + g$
2. fg
3. $g - f$
4. $f \circ g$
5. $g \circ f$
6. $(f \circ g)(3)$

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Elementary functions

Exercise 9

Graph each function. Give domain and range.

- Constant function $f(x) = c$ where c is any constant.
- Identity function $f(x) = x$
- Squaring function $f(x) = x^2$
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- Unit step function

$$f(x) = \begin{cases} 1, & \text{if } x \geq 0 \\ 0, & \text{Otherwise} \end{cases}$$

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Properties of functions

Let $c > 0$,

- The graph of $f(x+c)$ will be a shift of the graph of $f(x)$ to the left by c units.
- The graph of $f(x-c)$ will be a shift of the graph of $f(x)$ to the right by c units.
- The graph of $f(x)+c$ will be a shift of the graph of $f(x)$ upward by c units.
- The graph of $f(x)-c$ will be a shift of the graph of $f(x)$ downward by c units.
- The graph of $cf(x)(c > 1)$ will be a vertical stretch of the graph of $f(x)$ by a factor of c.

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Properties of functions

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- The graph of $f(cx)(c > 1)$ will be a horizontal compress of the graph of $f(x)$ by a factor of c.
- The graph of $f(cx)(0 < c < 1)$ will be a horizontal stretch of the graph of $f(x)$ by a factor of c.
- The graph of $-f(x)$ will be a reflection of the graph of $f(x)$ across the x-axis.
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Exercise 10

The graph of $f(x) = |x|; x \in [-3; 3]$, vertically stretched by a factor 2, reflected across the x-axis, shitted 5 units upward.

Give the equation, domain, range and graph of the resulting function.

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One-to-one functions

Definition

A function for which every element of the range of the function corresponds to exactly one element of the domain is called *a one-to-one function*.

- f is one-to-one if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$ for all x_1, x_2 in the domain of f.
- A function is one-to-one if no horizontal line intersects the graph of the function more than once.

Exercise 11

Determine each function is one-to-one.
(a) $f(x) = 3(x+1)^2$ (b) $g(x) = \sqrt{4x+1}$

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Inverse of a function

Definition

Let f and g be two one-to-one functions and the domain of f is $D(f)$, domain of g is $D(g)$, range of f is $R(f)$, and range of g is $R(g)$.

Suppose $D(g) = R(f)$ and $D(f) = R(g)$. Then, f and g are *inverse functions* of each other if $(f \circ g)(x) = (g \circ f)(x) = x$ for all x.

- The inverse of f is denoted by f^{-1}.
- If the ordered pair $(a; b)$ is on the graph of f, then (b, a) is on the graph of f^{-1}.
- The graph of f and f^{-1} are reflections across the line $y = x$:

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Exercise 12

Let $f(x) = \dfrac{2x-3}{x+4}$

Find the f^{-1}

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Find the f^{-1}

Odd and even functions

Definition

A function f is *even* if $f(-x) = f(x)$ for all x in the domain of f.

A function f is *odd* if $f(-x) = -f(x)$ for all x in the domain of f.

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- The graph of an even function is symmetric with respect to the y-axis.
- If (a, b) is on the graph of an even function then $(-a, b)$ is also on the graph.
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Exercise 13

Determine each function is even, odd or neither.

1. $f(x) = 3x^2 - 2$
2. $g(x) = 2x^3 + 5x$
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Periodic functions

Definition

A function f is called *periodic* if there is a positive constant T such that $f(x+T) = f(x)$ for all x in the domain of f. T is called the *period*. (If there exists a least positive constant P with this property, it is called the fundamental period.)

Exercise 14

An automated robot transports building materials between two locations, which are 6 km apart. It takes the robot two hours to reach the materials and two hours to return. Neglecting loading and unloading time, draw a displacement versus time graph that shows the robot performing 4 complete trips. Identify the period of this graph.

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Periodic functions

Definition

A function f is called *periodic* if there is a positive constant T such that $f(x+T) = f(x)$ for all x in the domain of f. T is called the *period*.
(If there exists a least positive constant P with this property, it is called the fundamental period.)

Exercise 14

An automated robot transports building materials between two locations, which are 6 km apart. It takes the robot two hours to reach the materials and two hours to return. Neglecting loading and unloading time, draw a displacement versus time graph that shows the robot performing 4 complete trips. Identify the period of this graph.

Outline

Categories of functions

- Linear functions
- Quadratic functions
- Polynomial functions
- Rational functions
- Exponential functions
- Logarithmic functions
- Hyperbolic functions
- Trigonometric functions

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Linear functions

Definition

A function f is *linear* if it has the form $f(x) = ax + b$ where a, b are constants $(a \neq 0)a$ is known as the slope of the function.

Exercise 15

Find the equation of a linear function f which passes through the points (2,-5) and (4, 7).

Graph f and locate x-intercept and y-intercept.

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Graph f and locate x-intercept and y-intercept.

Quadratic functions

Definition

A function f is *quadratic* if it has the form $f(x) = ax^2 + bx + c$ where a, b, c are constants ($a \neq 0$).

Exercise 16

Let $f(x) = 3x^2 - 7x - 2$: Complete square to write $f(x) = a(x - h)^2 + k$ form. Give the minimum value of f. Graph f and give zeros of f (A number c is a zero of a function if $f(c) = 0$).

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Polynomial functions

Definition

A function f is a polynomial of degree n if it has the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + .. + a_0$ where $a_n, a_{n-1}, ..., a_0$ are constants $(a_n \neq 0)$ and n $\in \mathbb{N}$.

Exercise 17

Let $f(x) = 3(x+1)^3(2x-7)^2(x-10)$. Graph f.

Solve the inequality $f(x) > 0$.

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Rational functions

Definition

A function f is *rational* if it has the form $f(x) = \frac{p(x)}{q(x)}$ (where p and q are polynomials and $q(x) \neq 0$)

- The line $x = a$ is called a *vertical asymptote* if f approaches $\pm\infty$ as x approaches a.
- The line $y = b$ is called a *horizontal asymptote* if f approached b as x approached $\pm\infty$.

Exercise 18

Let $f(x) = \frac{x^2-3x+2}{(2x-5)(x+5)}$. Give domain of f. Graph f.

Find vertical asymptotes and horizontal asymptotes of f.

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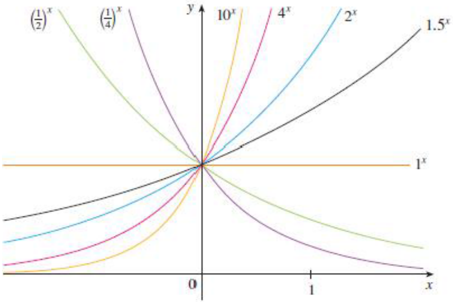
Exponential function

Definition

A function f is exponential if it has the form $f(x) = ax$ where $a > 0, a \neq 1$ is a constant called the *base* of f. (When a = e it is called the natural exponential function.)



Exercise 19

Graph

- $f(x) = 2^{1-x}$
- $g(x) = 3 - (\frac{1}{2})^x$
- $h(x) = 3 \cdot 2^{x-3}$

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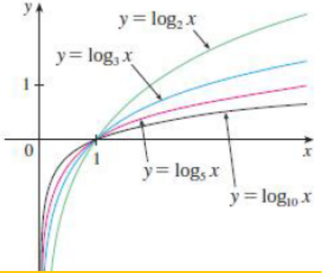
Logarithmic functions

Definition

If $x = a^x$ then $y = log_a$ x is called the *logarithmic function* of base $a(a > 0, a \neq 1)$.

(When $a = e$ it is written ln x and called the natural logarithmic function.)



- The functions $f(x) = a^x$ and $g(x) = log_a x$ are inverse functions of each other.

Exercise 20

Graph

1. $f(x) = log(x + 1)$
2. $g(x) = log \mid x - 1 \mid$
3. $h(x) = 2 - ln(2x - 3)$

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Exercise 21: Solve

- $2ln\sqrt{x} - ln(1-x) - 2 = 0$
- $e^{ln(x+1)} - xe^{3x+5} = 1$

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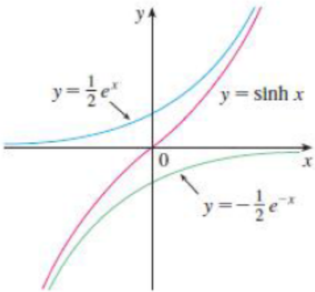
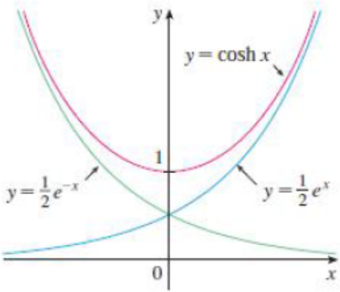
Hyperbolic functions

Definition

The *hyperbolic functions* are defined as $coshx = \frac{1}{2}(e^x + e^{-x})$ and $sinhx = \frac{1}{2}(e^x - e^{-x})$





Exercise 22

Graph *coshx* and *sinhx*. Give their domain and range and comment on even or oddness.

- Other hyperbolic functions are defined as:

$$tanhx = \frac{sinhx}{coshx},$$

$$cschx = \frac{1}{sinhx},$$

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Exercise 23

Prove the following identities.

- $cosh^2 x - sinh^2 x = 1$ for all x.
- $sinh(-x) = -sinh(x)$
- $cosh(-x) = cosh(x)$
- $1 - tanh^2 x = sech^2 x$
- $sinh(x + y) = sinhxcoshy + coshxsinhy$
- $cosh(x + y) = coshxcoshy + sinhxsinhy$

Exercise

Express $2e^x - e^{-x}$ in terms of $coshx$ and $sinhx$.

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Exercise

Express $2e^x - e^{-x}$ in terms of $coshx$ and $sinhx$.

Exercise: Show that

- $sinh^{-1}x = ln(x + \sqrt{x^2 + 1})$
- $cosh^{-1}x = ln(x + \sqrt{x^2 - 1})$
- $tanh^{-1} = \frac{1}{2}ln(\frac{1 + x}{1 - x})$

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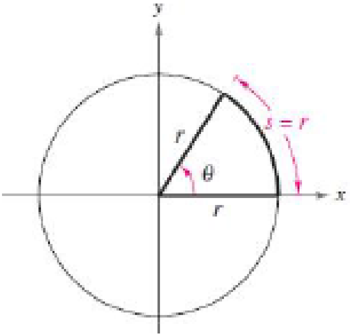
Trigonometric functions

Definition

One radian (rad) is the measure of a central angle that intercepts an arc s equal in length to the radius r of the circle. I.e., $\theta = \frac{s}{r}$ and, π rad $= 180^0$.



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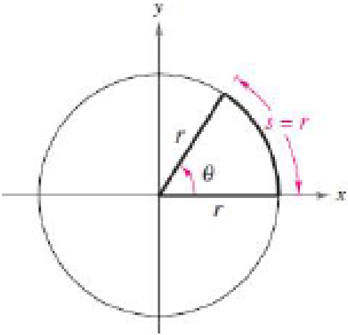
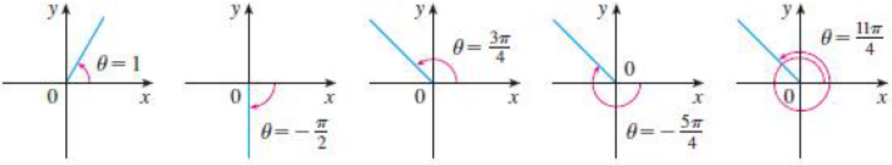


Exercise 26

For each angle in standard position, identify initial side, terminal side, angles measured counterclockwise or clockwise direction.



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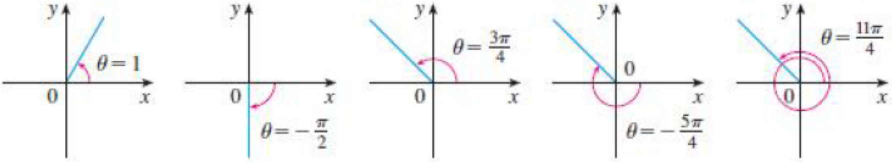
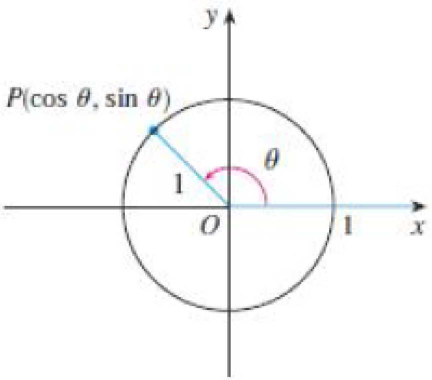
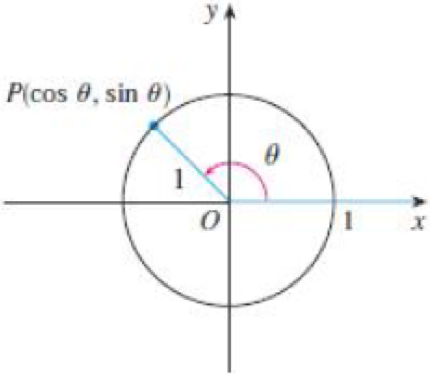


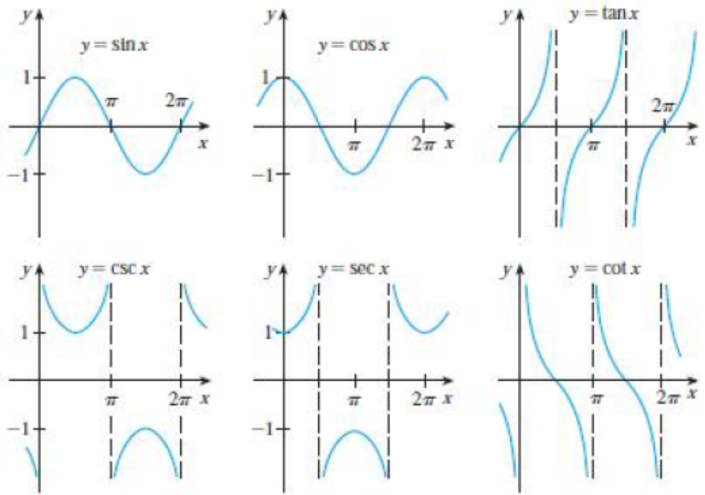


Let P be on the unit circle, centered at origin. Define the following ratios as follows whenever they make sense.



Let P be on the unit circle, centered at origin. Define the following ratios as follows whenever they make sense.

$$sin\theta = y, cos\theta = x, tan\theta = \frac{y}{x}\ csc\theta = \frac{1}{y}, sec\theta = \frac{1}{x}, cot\theta = \frac{x}{y}$$



Exercise 27

Graph each function and give its domain, range, and period.

- $f(x) = sinx$
- $f(x) = cosx$
- $f(x) = tanx$
- $f(x) = cscx$
- $f(x) = secx$
- $f(x) = cotx$
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