Graph Theory and Its Applications

Dr. G.H.J. Lanel

Lecture 7

Outline

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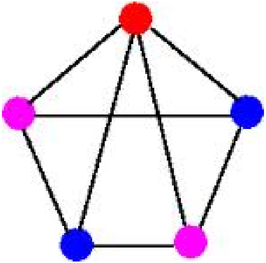
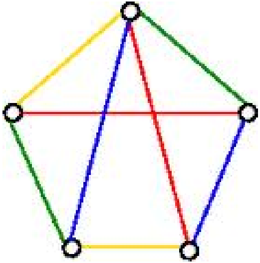
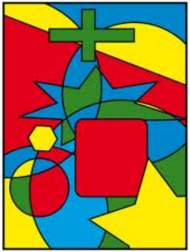
Example: Vertex coloring



Example: Edge coloring



Example: Face coloring



Definition

Let $G = (V, E)$ be a graph. A k-coloring of G is a function $C : V \longrightarrow \{1, 2, \cdots, k\}$ such that $C(u) \neq C(v)$ whenever $uv \in E$.

Remark:

1. A graph that has a k-coloring is said to be k-colorable.
2. Any graph that is k-colorable is also k'-colorable for all $k' > k$.

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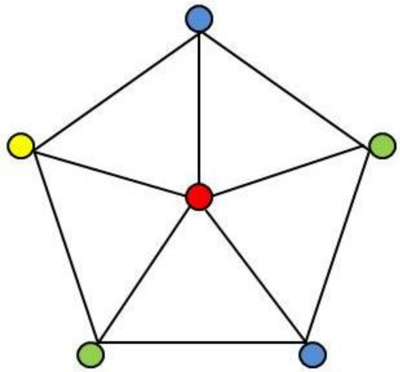
The minimum value of k for which there exists a k-coloring of G is called the chromatic number of G, and it is denoted by ($\chi(G)$).

Thus G is k-colorable if and only if $\iff \chi(G)) \leq k$.

Question: Prove that $\chi(G)) = 4$ for the following graph.

Proof: Follows...



Theorem

A graph is bipartite if and only if its chromatic number is at most 2.

Proof

Homework.

Theorem

If K_n is a subgraph of a graph G, then $\chi(G)) \geq n$.

Proof

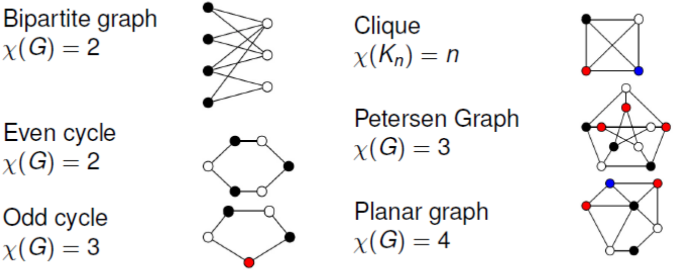
***Proof**: Follows...*

Theorem

The chromatic number of the n-cycle is given by,

$$\chi(n) = \begin{cases} 2 & \textit{if } n \textit{ is even} \\ 3 & \textit{if } n \textit{ is odd} \end{cases}$$

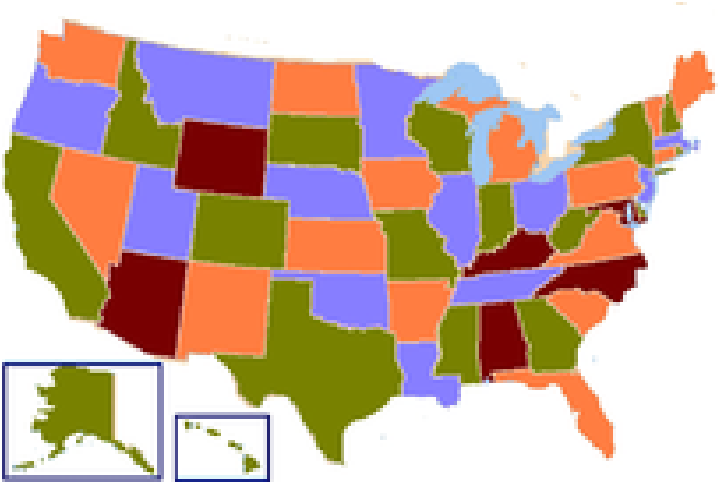
Some examples of known chromatic numbers are :



Outline

Theorem

If G is a planar graph, then $\chi(n) \leq 4$



Outline

1. Graph Coloring
2. Chromatic number ($\chi(G)$)
3. Four color theorem
4. Coloring Algorithm

- Finding an optimal coloring is exhaustive search.
- Start with one node, give it a color, assign non-conflicting colors to its neighbors, and so on.
- Try it with two colors, if you get no result, then try three, and so on.
- They are a lot of heuristic algorithms (not proven mathematically it is the best) that try to improve on that both by reducing space and time

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Greedy Algorithm

Algorithm GreedyColor(G)

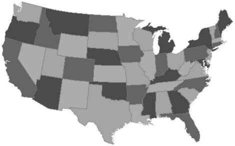
L := sort(V); c := sort(colors)

for $v \in V$ **do**

choose smallest c_i not used by colored neighbors

end for

Let us come back to the map coloring problem



and try to prove the following (simpler) result

Assignment #3: Every planar graph can be colored with 6 colors.

Show that $e \leq 3n - 6$,

Show then that for planar graphs $average(d(v)) \leq 6 - 12 / n$,

Finally prove that there exists a v such that $d(v) \leq 5$,

Now use induction to prove the proposition (remove nodes).