Graph Theory and Its Applications

Dr. G.H.J. Lanel

Lecture 8

Outline

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A Problem

- A tropical fish hobbyist had six different types of fish: Alphas, Betas, Certas, Deltas, Epsalas, and Fetas, which shall henceforth be designated by A, B, C, D, E, and F, respectively.
- Because of predator-prey relationships, water conditions, and size, some fish can be kept in the same tank. The following table shows which fish cannot be together:

Type	A	B	C	D	E	F
Can not be with	B,C	A,C,D	A,B,D,E	B,C,F	C,F	D,E

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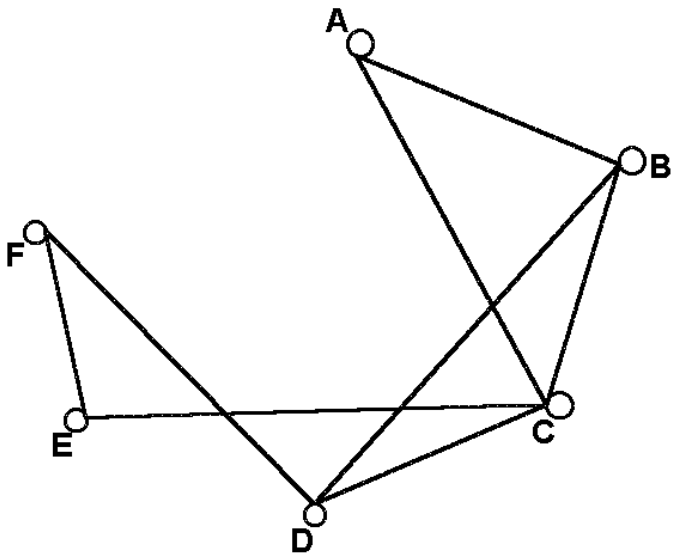
A Graph Theoretical Model

Question: What is the smallest number of tanks needed to keep all the fish?

- We will use a graph to help us answer this question.
- Below you will see an uncolored graph that describes this situation.
- Can you figure out what each vertex represents? each edge?



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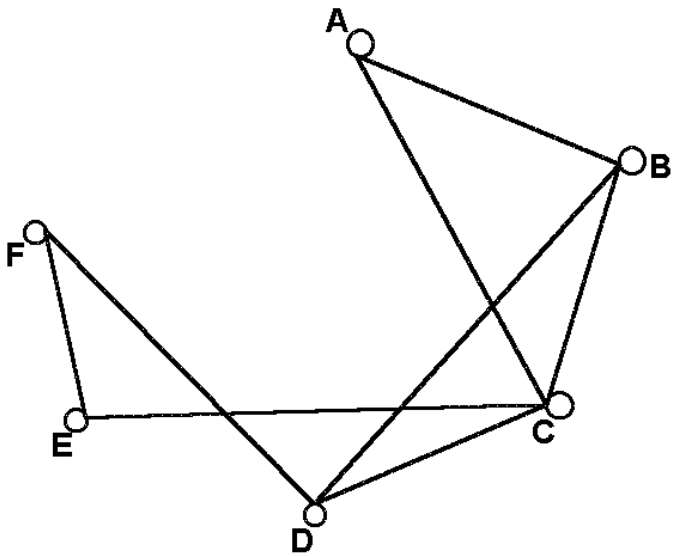

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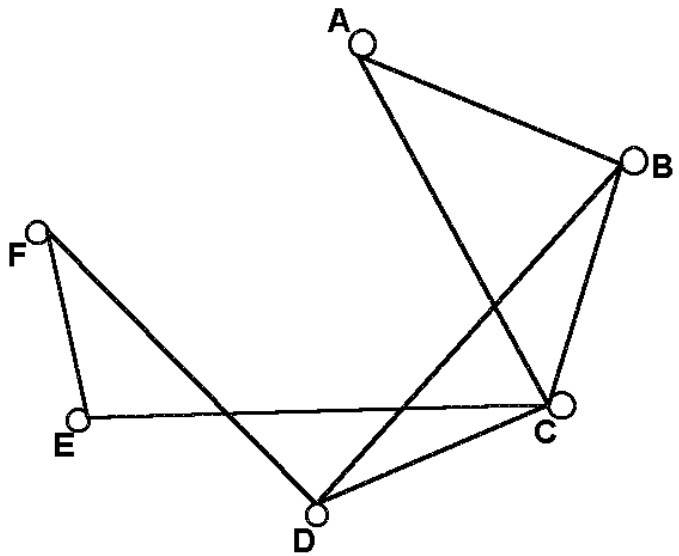

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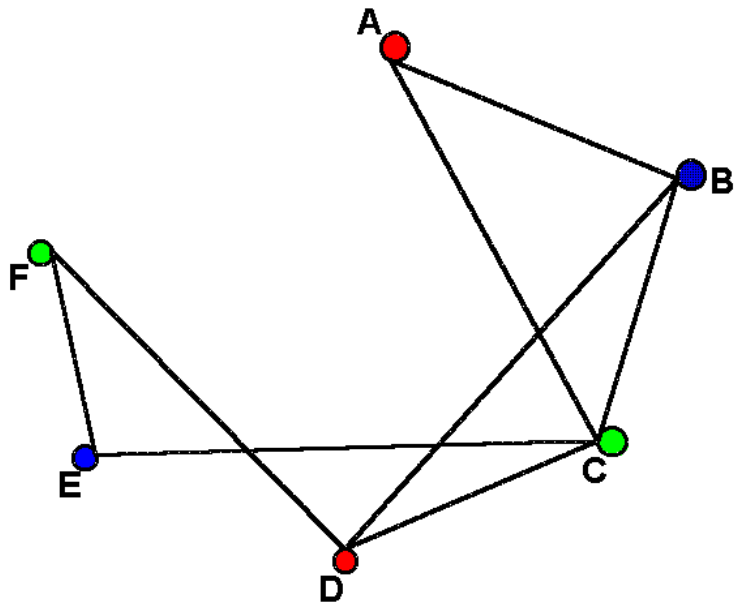
- So if each vertex represents one of the types of fish and each edge connects vertices that are not compatible,
- What does each color on the graph represent?
- How does the chromatic number help you to solve this problem?
- Here is the graph again now with color.



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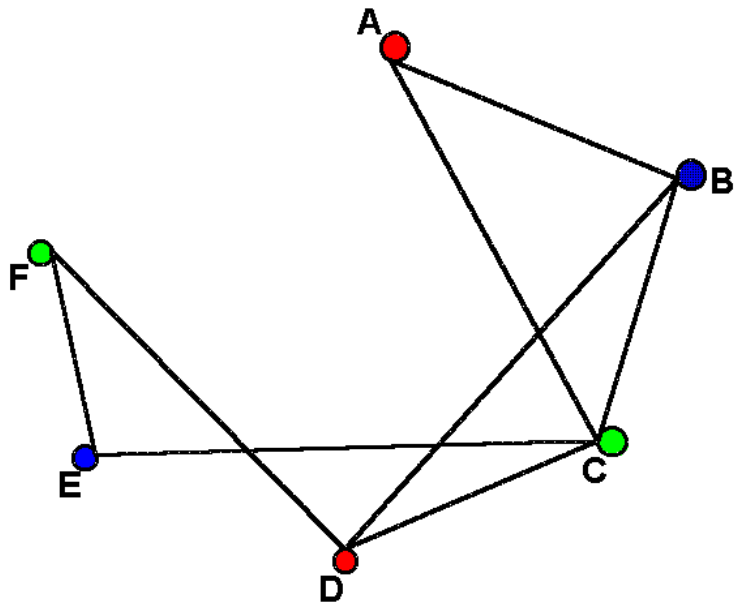

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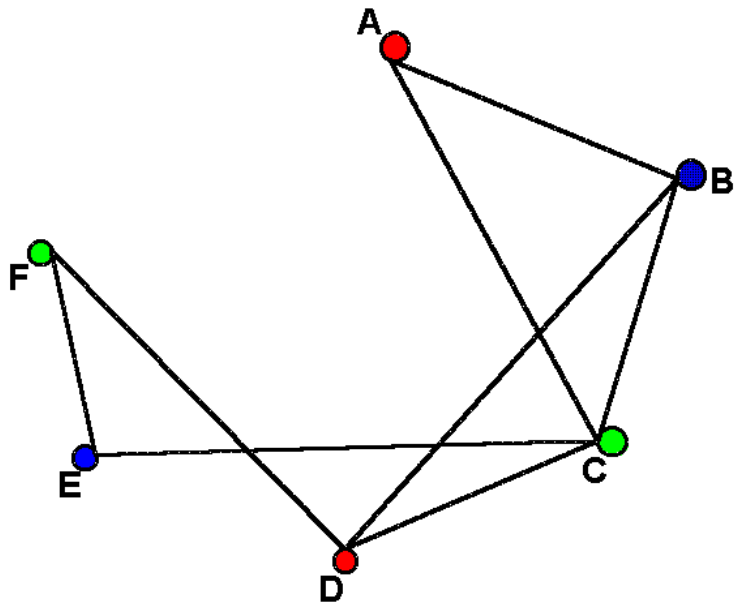

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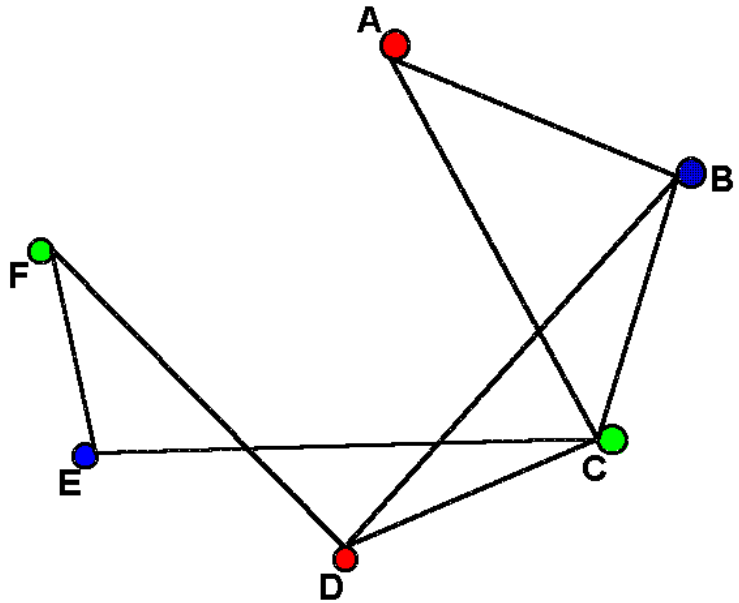

The Solution

- The fewest number of tanks the tropical fish owner will need is three.
- Several different combinations of fish are possible depending on how the graph is colored.
- Below is the solution for how our graph was colored. Note that fish with vertices of the same color go into the same tank.

Tank 1	Tank 2	Tank 3
Alphas and Deltas	Fetas and Certas	Betas and Epsalas

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- Suppose that we want to schedule some final exams for MAT courses with following course numbers:

  1, 2, 3, 4, 5, 6, 7 and 8.

- Suppose also that there are no students in common taking the following pairs of courses:

  1-2, 1-3, 1-4, 1-5, 1-6, 1-7, 1-8
  2-3, 2-5, 2-6, 2-7, 2-8
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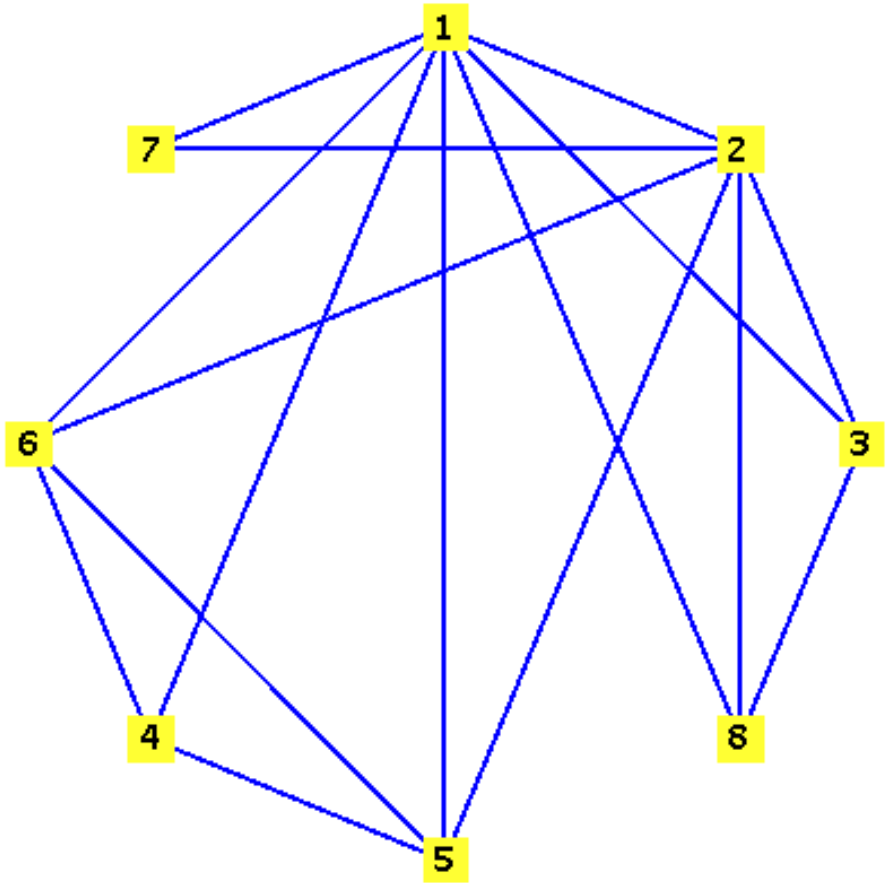
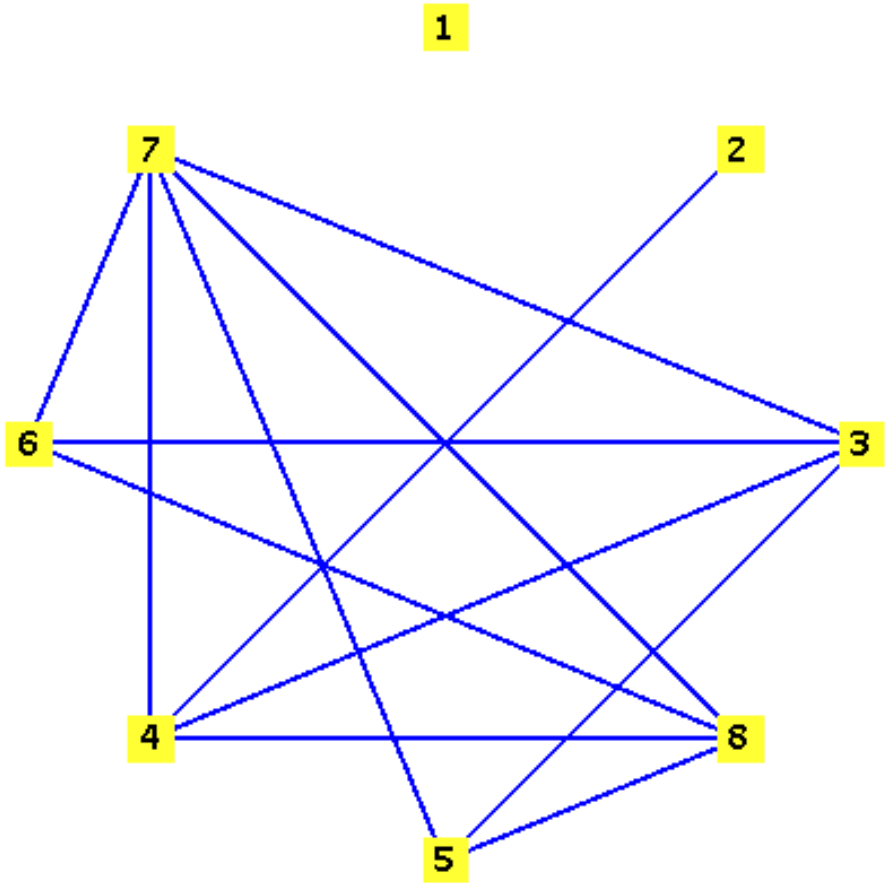
Graph



The Complementary Graph



The Solution

- The graph is obviously not 1-colorable because there exist edges.
- The graph is not 2-colorable because there exist triangles.
- Is it 3-colorable? Try to color by Red, Green, Blue.

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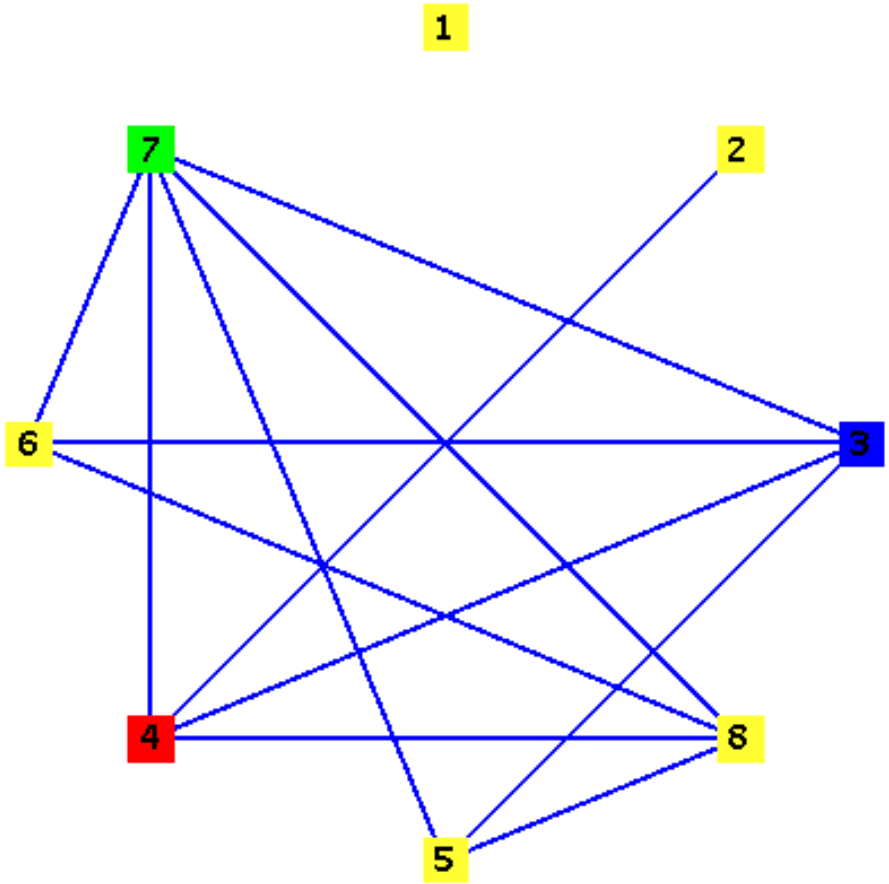
The Solution

Pick a triangle and color the vertices 3-Blue, 4-Red and 7-Green.



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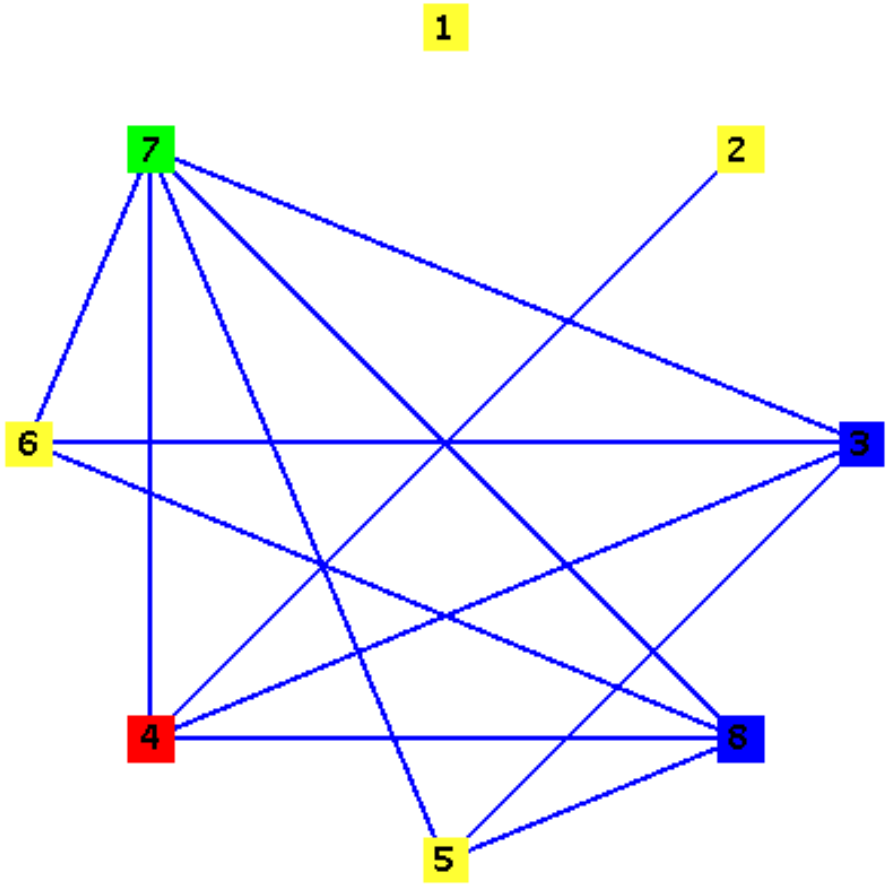
So the vertex 8 must be blue.



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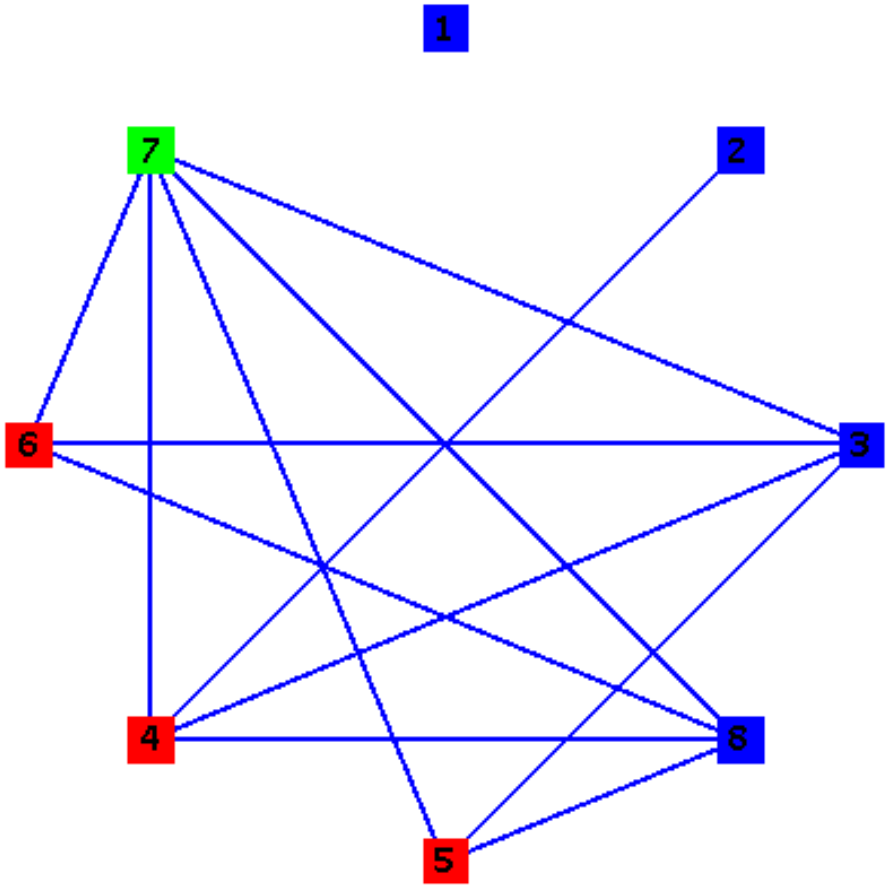
Then the vertices 5 and 6 must be red. For the 1 and 2 we pick blue.



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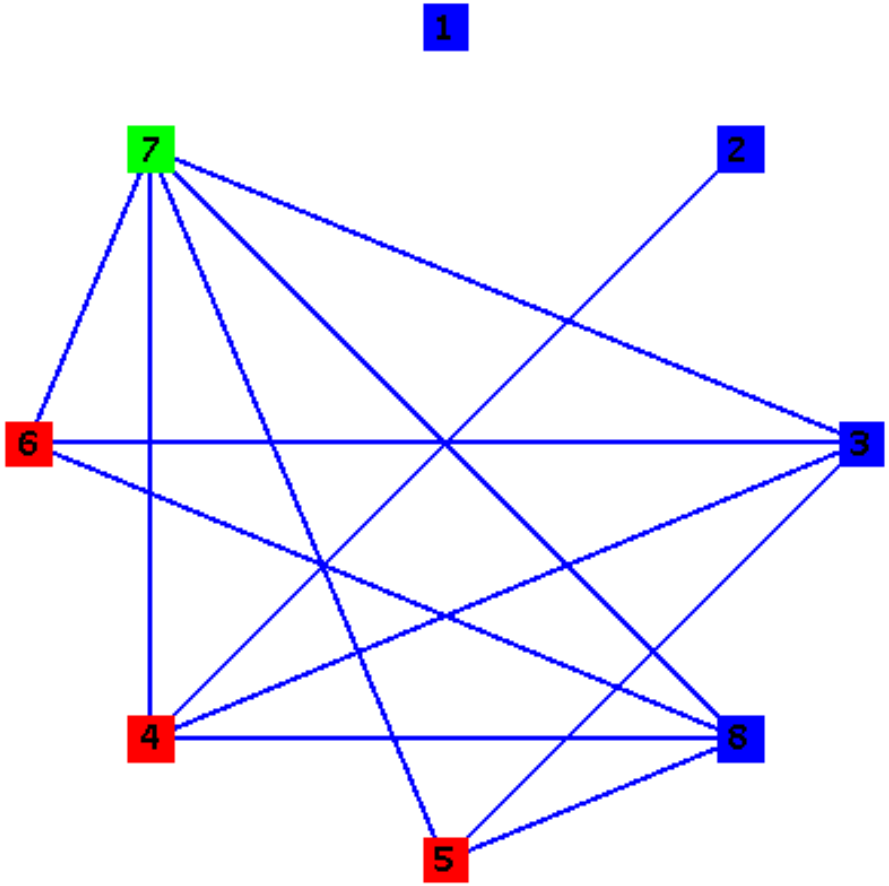
Therefore we need three exam slots, which is given by three colors.



- Twelve faculty members in a mathematics department serve on the following committees:

  Undergraduate education: SI, LI, AX, FU

  Graduate Education: GR, VE, FU, IN

  Colloquium: LE, RA, PR

  Library: VA, SI, LE

  Staffing: GR, RA, VE, LI

  Promotion: VE, VA, PA

- The committees must all meet during the first week of classes, but there are only three time slots available.

- Find a schedule that will allow all faculty members to attend the meetings of all committees on which they serve.

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  Math, English, Computer Science, Geography

  Biology, Psychology, Geography, Spanish

  Biology, Computer Science, History, French

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- What is the minimum number of examination periods required for the exams in the ten courses specified so that students taking any of the given combinations of courses have no conflicts?
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