Trees

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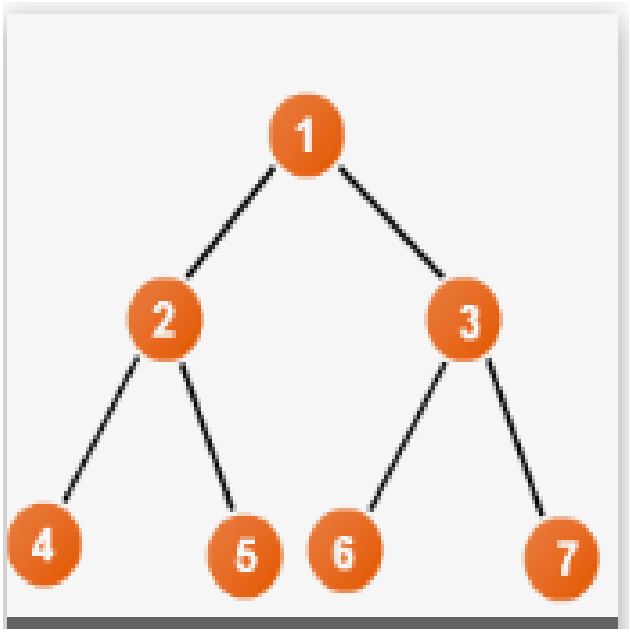
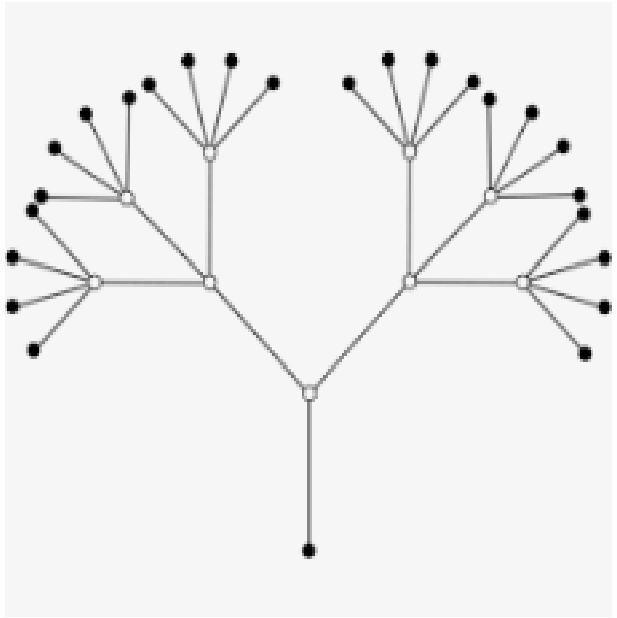
Outline

Tree

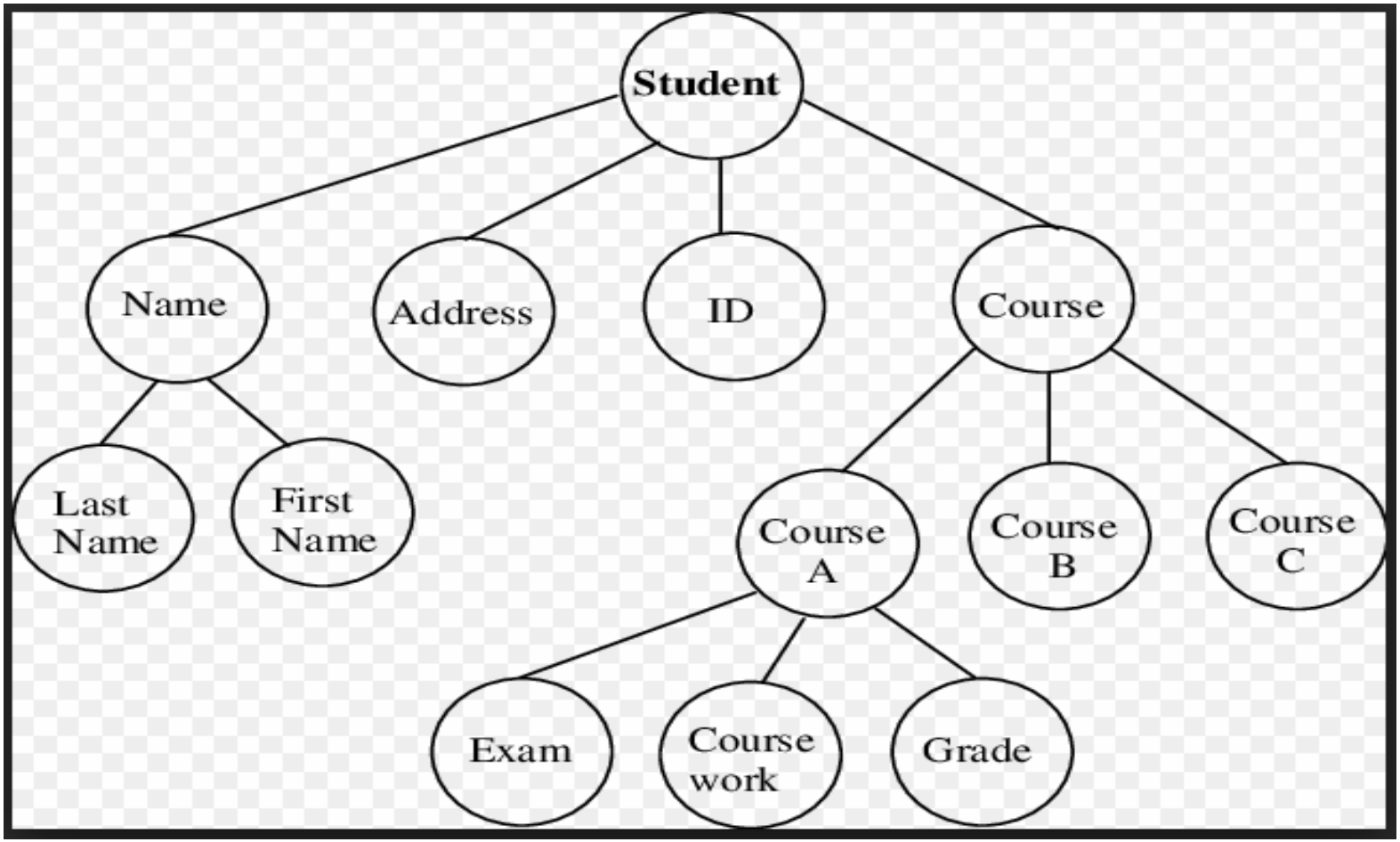
A tree T is a connected graph that has no cycles.





[Mapple command; G:=Graph(,); DrawGraph(G,style=tree, root=v)]

An example of a tree data structure



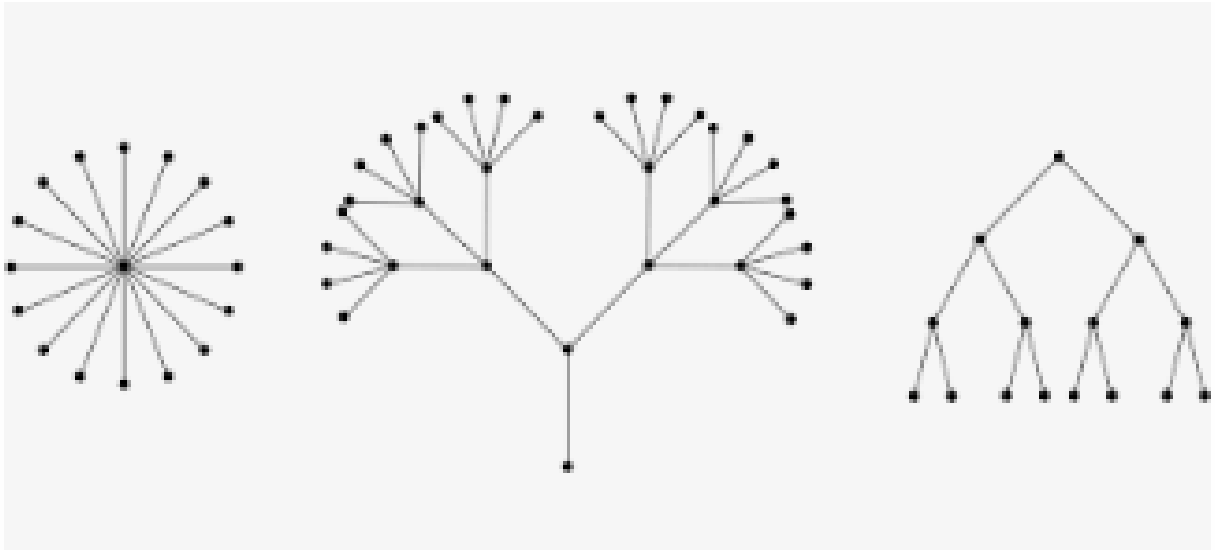
Definition

In an undirected tree, a *leaf* is a vertex of degree 1. [**Maple command IsTree**]

Definition

An acyclic graph is called a *forest*. [**Maple command IsForest**]

Example:



A forest G on n vertices has $n - c(G)$ edges, where $c(G)$ is the number of components of G.(True for any graph)

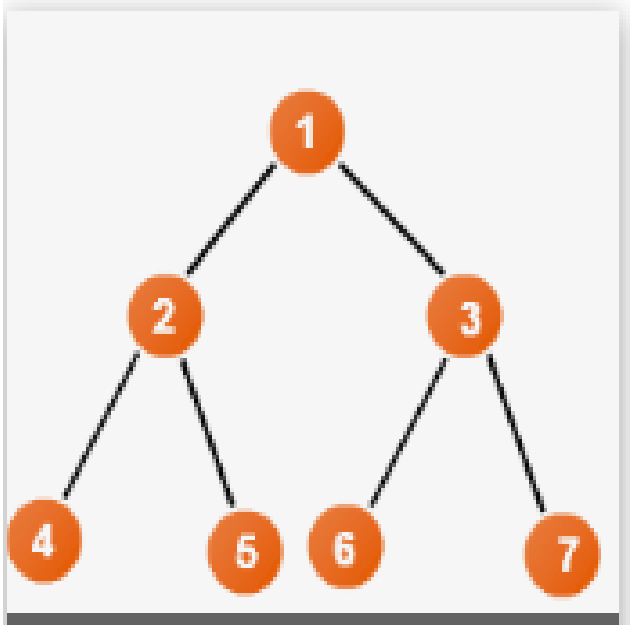
Proposition

For a graph $G = (V, E)$ of order $n = |V|$, the following are equivalent

- G is a Tree.
- $\forall u, v \in V$ there is one and only one path from u to v.
- G is connected and has $n - 1$ edges.
- G is acyclic and has $n - 1$ edges.
- G is acyclic and adding an edge creates one and only one cycle.

Rooted tree

A tree is called a *rooted tree* if one vertex has been designated the root, in which case the edges have a natural orientation, towards or away from the root.



- In a rooted tree, the *depth or level* of a vertex v is its distance from the root.
- The *height* of a rooted tree is the length of a longest path from the root.(Maple command has different meaning for this)
- If vertex v immediately precedes vertex w on the path from the root to w, then v is *parent* of w and w is *child* of v.
- Vertices having the same parent are called *siblings*.
- A *leaf* in a rooted tree is any vertex having no children.

Maple commands

- Spanning Tree
  The *SpanningTree* command returns a spanning tree of G, a subgraph that contains all the vertices and is a tree.

- Tree Height
  TreeHeight returns the height of the tree T with the vertex r as root. In other words it returns the maximum distance of the vertices of T from r.

Maple commands

- Shortest Path
  ShortestPath returns a shortest path from u to v using a breadth-first search. Edge weights are ignored. The output is a list of vertices in the order they appear on the path. If no such a path exists, an error is returned.

- Distance
  Distance returns the number of edges in the shortest path from s to t. If no such path exists, the output is infinity. The strategy is to use a breadth-first search (BFS).