

# Graph Theory and Its Applications

Dr. G.H.J. Lanel

Lecture 8

# Outline

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## 1 Applications of Graph Coloring

- Sorting Fish
- Final Exam Scheduling
- Committees Scheduling
- Another Application on Exam Scheduling

# A Problem

- A tropical fish hobbyist had six different types of fish: Alphas, Betas, Certas, Deltas, Epsalas, and Fetas, which shall henceforth be designated by A, B, C, D, E, and F, respectively.
- Because of predator-prey relationships, water conditions, and size, some fish can be kept in the same tank. The following table shows which fish cannot be together:

Type	A	B	C	D	E	F
Can not be with	B,C	A,C,D	A,B,D,E	B,C,F	C,F	D,E

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Question: What is the smallest number of tanks needed to keep all the fish?

- We will use a graph to help us answer this question.
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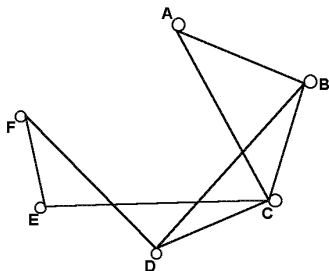
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- What does each color on the graph represent?
- How does the chromatic number help you to solve this problem?
- Here is the graph again now with color.

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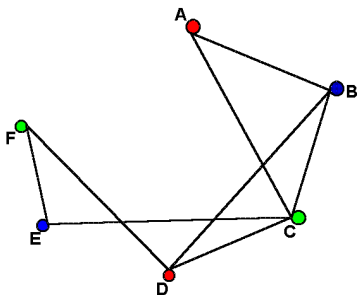
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# The Solution

- The fewest number of tanks the tropical fish owner will need is three.
- Several different combinations of fish are possible depending on how the graph is colored.
- Below is the solution for how our graph was colored. Note that fish with vertices of the same color go into the same tank.

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Tank 1	Tank 2	Tank 3
Alphas and Deltas	Fetas and Certas	Betas and Epsalas

# A Problem

- Suppose that we want to schedule some final exams for MAT courses with following course numbers:

1, 2, 3, 4, 5, 6, 7 and 8.

- Suppose also that there are no students in common taking the following pairs of courses:

1-2, 1-3, 1-4, 1-5, 1-6, 1-7, 1-8

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4-5, 4-6

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- Courses are represented by vertices.
- Two vertices are connected with an edge if the corresponding courses have no students in common.

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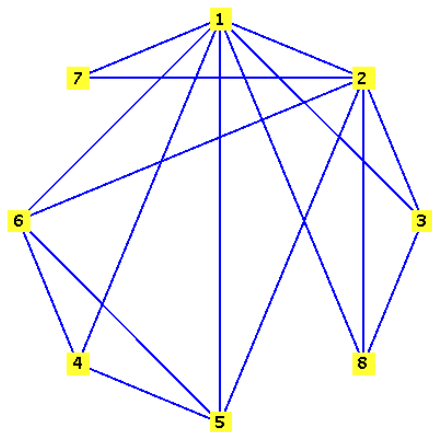
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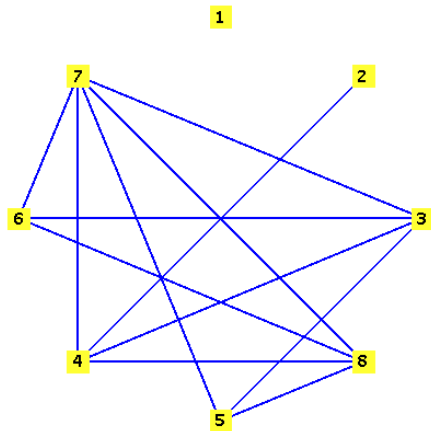
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# Graph



# The Complementary Graph



# The Solution

- The graph is obviously not 1-colorable because there exist edges.
- The graph is not 2-colorable because there exist triangles.
- Is it 3-colorable? Try to color by Red, Green, Blue.

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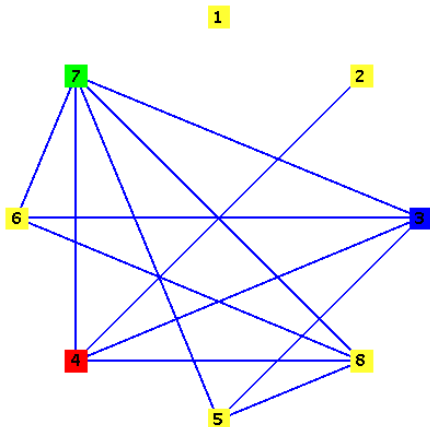
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Pick a triangle and color the vertices 3-Blue, 4-Red and 7-Green.

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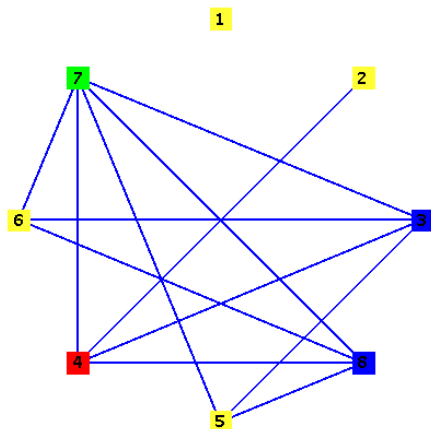
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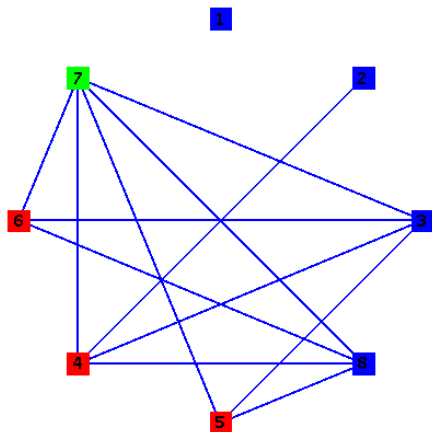


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Then the vertices 5 and 6 must be **red**. For the 1 and 2 we pick **blue**.

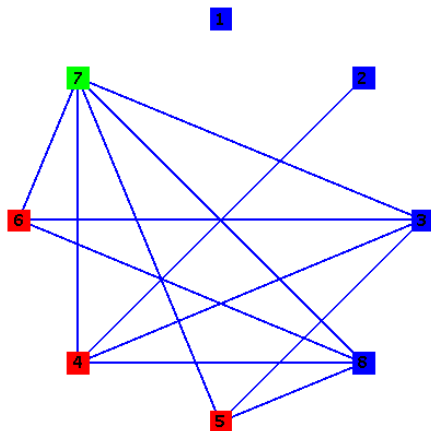
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Therefore we need three exam slots, which is given by three colors.



- Twelve faculty members in a mathematics department serve on the following committees:

**Undergraduate education:** SI, LI, AX, FU

**Graduate Education:** GR, VE, FU, IN

**Colloquium:** LE, RA, PR

**Library:** VA, SI, LE

**Staffing:** GR, RA, VE, LI

**Promotion:** VE, VA, PA

- The committees must all meet during the first week of classes, but there are only three time slots available.
- Find a schedule that will allow all faculty members to attend the meetings of all committees on which they serve.

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- Suppose that in a particular quarter there are students taking each of the following combinations of courses:

Math, English, Biology, Chemistry

Math, English, Computer Science, Geography

Biology, Psychology, Geography, Spanish

Biology, Computer Science, History, French

English, Psychology, Computer Science, History

Psychology, Chemistry, Computer Science, French

Psychology, Geography, History, Spanish

- What is the minimum number of examination periods required for the exams in the ten courses specified so that students taking any of the given combinations of courses have no conflicts?
- Find a schedule that uses this minimum number of periods.



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