MA1302 Engineering Mathematics I

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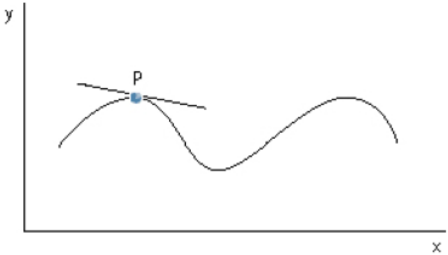
Lecture 1-Differentiation

Outline

- Differentiation allows us to find rates of change.
- For example, it allows us to find the rate of change of velocity with respect to time (which is acceleration).
- It also allows us to find the rate of change of x with respect to y, which on a graph of y against x is the gradient of the curve(The slope of the tangent line is equal to the derivative of the function at the marked point).



- There are a number of simple rules which can be used to allow us to differentiate many functions easily.
- If y = some function of x (in other words if y is equal to an expression containing numbers and x's), then the derivative of y (with respect to x) is written $\frac{dy}{dx}$, pronounced ”dee y by dee x”.
- This is also known as ’Leibniz Notation’.

There are many ways a question can ask you to differentiate:

- Differentiate the function...
- Find $f^{'}(x)$
- Find $\frac{\mathrm{d}y}{\mathrm{d}x}$
- Calculate the rate of change of...
- Find the derivative of...
- Calculate the gradient of the tangent to the curve

Outline

We considered the derivative of a function f at a fixed number a:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad (1)$$

Here we change our point of view and let the number a vary. If we replace a in equation by a variable x, we obtain

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad (2)$$

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$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad (2)$$

- Given any number x for which this limit exists, we assign to x the number $f'(x)$.
- So we can regard f' as a new function, called the derivative of f and defined by equation (2).
- We know that the value of f' at x, can be interpreted geometrically as the slope of the tangent line to the graph of f at the point $(x, f(x))$.
- The function f' is called the derivative of f because it has been derived from f by the limiting operation in equation (2).
- The domain of f' is the set $\{x | f'(x) \textit{ exists}\}$ and may be smaller than the domain of f.

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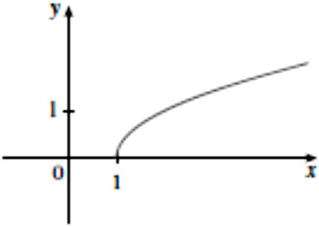
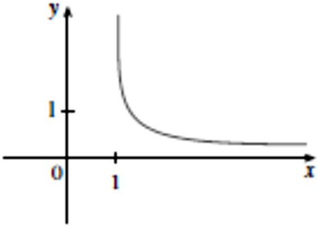
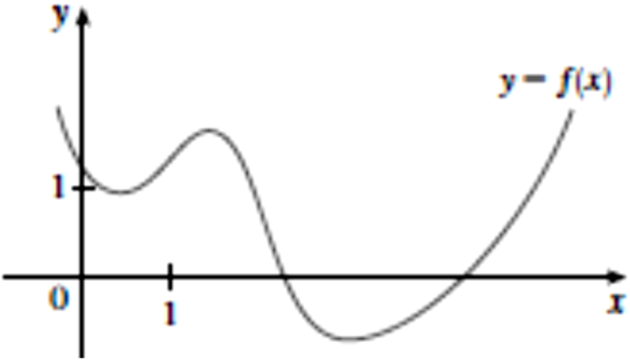


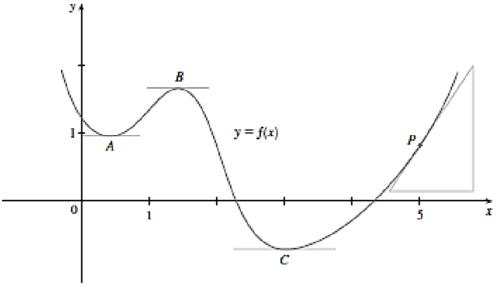
(a) $f(x)=\sqrt{x-1}$



(b) $f'(x)=\dfrac{1}{2\sqrt{x-1}}$

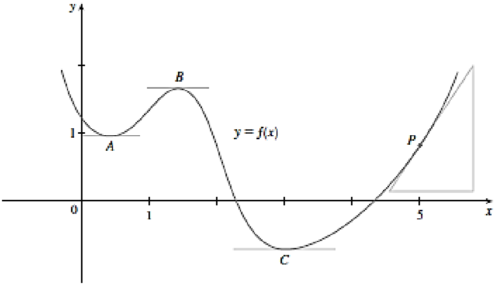


Sketch the graph of the derivative f'.



(a)

- Tangents at A, B and C are horizontal, so the derivative is 0 there, and the graph of f' crosses the x-axis at the points A', B' and C'..
- Between A and B the tangents have positive slope, so $f'(x)$ is positive there.
- But between B and C the tangents have negative slope, so $f'(x)$ is negative there.



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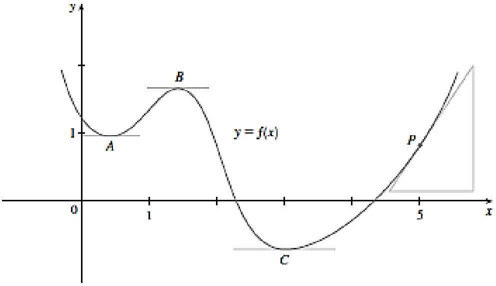

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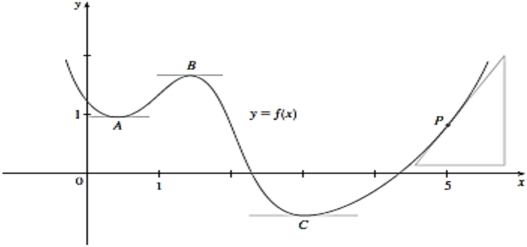

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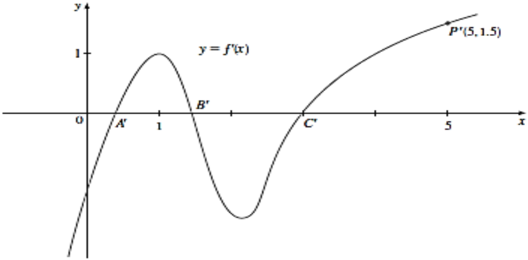

(b)

If we use the traditional notation $y = f(x)$ to indicate that the independent variable is x and the dependent variable is y, then some common alternative notations for the derivative are as follows:

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}f(x) = Df(x) = D_x f(x)$$

The symbols D and $\frac{d}{dx}$ are called differentiable operators because they indicate the operation of differentiation, which is the process of calculating a derivative.

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Second derivative

If f is a differentiable function, then its derivative f' is also a function, so f' may have a derivative of its own, denoted by $(f')' = f''$. This new function $f''$is called the **second derivative** of because it is the derivative of the derivative of f . We write the second derivative of $y = f(x)$ as

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Example:

If $f(x) = x^3 - x$, find an interpret $f''(x)$.

Solution:

first derivative is$f'(x) = 3x^2 - 1$. So the second derivative is;

$f''(x) = (f')'(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h} = \lim_{h \to 0} \frac{[3*(x+h)^2 - 1] - [3x^2 - 1]}{h} = \lim_{h \to 0} \frac{3x^2 + 6xh 3h^2 - 1 - 3x^2 + 1}{h} = \lim_{h \to 0} \frac{6x + 3h}{h} = 6x$

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Third derivative

Third derivative is the derivative of the second derivative: $f''' = (f'')'$. So $f'''(x)$ can be interpreted as the slope of the curve $y = f''(x)$ or as the rate of change of $f''(x)$. If $y = f(x)$, then alternative notations for the third derivative are,

$$y''' = f'''(x) = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3}$$

The process can be continued. The fourth derivative f'''' is usually denoted by $f^{(4)}$. In general, the nth derivative of is denoted by $f^{(n)}$ and is obtained from f by differentiating n times. If $y = f(x)$, we write

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Outline

- Derivative of a constant function is zero

$$\frac{\mathrm{d}c}{\mathrm{d}x} = 0$$

- Constant multiple rule

$$\frac{\mathrm{d}cf(x)}{\mathrm{d}x} = c\frac{\mathrm{d}f(x)}{\mathrm{d}x}$$

- Power rule

$$\frac{\mathrm{d}x^n}{\mathrm{d}x} = nx^{n-1}$$

Example : Differentiate $y = (4x - 3)^5$.

Basic standard form is $y = x^5$, $\frac{dy}{dx} = 5x^4$

Here , $(4x - 3)$ replaces the single x.

Hence $\frac{dy}{dx} = 5(4x - 3)^4 X$ the diff. of the function $(4x - 3)$

$= 5(4x - 3)^4 X 4 = 20(4x - 3)^4$.

Therefore $\frac{dy}{dx} = 20(4x - 3)^4$

Exercise 1 : Differentiate

1. $f(x) = 2x^3 - 4x^2 + 5x - 3$
2. $g(x) = \frac{(x-1)(2x+3)}{A}$ (A is a constant)
3. $y = x^{\frac{3}{2}} - \frac{4}{3}x^{\frac{-3}{4}}$
4. $y = (x + x^{-1})^4$

- Sum rule

  $(f+g)' = f' + g'$

- Difference rule

  $(f-g)' = f' - g'$

- Product rule $(f.g)' = f.g' + f'g$

- Quotient rule $(\frac{f}{g})' = \frac{g.f' - f.g'}{g^2}$

- Chain rule

  $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ where $y = f(u)$ and $u = g(x)$

- **Exponential functions**
- $\frac{\mathrm{d}a^x}{\mathrm{d}x} = a^x(ln\, a)$
- $\frac{\mathrm{d}e^x}{\mathrm{d}x} = e^x$
- **Logarithmic functions**
- $\frac{\mathrm{d}\log_a x}{\mathrm{d}x} = \frac{1}{x(lna)}$
- $\frac{\mathrm{d}\ln|x|}{\mathrm{d}x} = \frac{1}{x}$

Example : Differentiate $y = e^{3-x}$

$$\begin{aligned} y &= e^{3-x} \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= e^{3-x}(-1) = -e^{3-x} \end{aligned}$$

Example : Differentiate $y = \log_{10}(2x - 1)$.

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{1}{(2x-1)\ln 10}.2 \\ &= \frac{2}{(2x-1)\ln 10} \end{aligned}$$

Exercise 2 : Differentiate

1. $f(x) = (x^2 + 3x)e^x$
2. $y = \frac{2x+1}{x^2-3x+5}$
3. $g(x) = (1 + xe^x)(1 - e^x)^{-1}$
4. $Z = \frac{w}{w+\frac{k}{w}}$ where k is a constant.
5. $y = (1 + e^x)(x + e^x)$

Exercise 3 : Differentiate

1. $f(x) = \ln \frac{1}{x} + \frac{1}{\ln x}$
2. $y = \log_3(xe^x)$
3. $G(y) = \ln|\cos(\ln x)|$
4. $y = \ln \ln \ln s$
5. $F(v) = \frac{\log_3 3v}{1+\log_5 5v}$

■ Trigonometric functions.

- $\frac{\mathrm{d} \sin x}{\mathrm{d} x} = \cos x$
- $\frac{\mathrm{d} \cos x}{\mathrm{d} x} = -\sin x$
- $\frac{\mathrm{d} \tan x}{\mathrm{d} x} = \sec^2 x$
- $\frac{\mathrm{d} \csc x}{\mathrm{d} x} = -\csc x \cot x$
- $\frac{\mathrm{d} \sec x}{\mathrm{d} x} = \sec x \tan x$
- $\frac{\mathrm{d} \cot x}{\mathrm{d} x} = -\csc^2 x \tan x$

Inverse trigonometric functions

- $\frac{\mathrm{d}\sin^{-1}x}{\mathrm{d}x} = \frac{1}{\sqrt{1-x^2}}$
- $\frac{\mathrm{d}\cos^{-1}x}{\mathrm{d}x} = -\frac{1}{\sqrt{1-x^2}}$
- $\frac{\mathrm{d}\tan^{-1}x}{\mathrm{d}x} = \frac{1}{1+x^2}$
- $\frac{\mathrm{d}\csc^{-1}x}{\mathrm{d}x} = -\frac{1}{x\sqrt{x^2-1}}$
- $\frac{\mathrm{d}\sec^{-1}x}{\mathrm{d}x} = \frac{1}{x\sqrt{x^2-1}}$
- $\frac{\mathrm{d}\cot^{-1}x}{\mathrm{d}x} = -\frac{1}{1+x^2}$

- Inverse rule

$$\left[f^{-1(x)}\right]' = \frac{1}{f'(f^{-1}(x))}$$

- Parametric differentiation

$$\text{If} x = h(t) \quad \text{and} \quad y = g(t) \quad \text{then} \quad \frac{\mathrm{d}y}{\mathrm{d}x} = (\frac{\mathrm{d}y}{\mathrm{d}t})/(\frac{\mathrm{d}x}{\mathrm{d}t})$$

Example : Differentiate $y = \frac{\sin 3x}{x+1}$

$$\begin{aligned} y &= \frac{\sin 3x}{x+1} \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{3(x+1)\cos 3x - \sin 3x.1}{(x+1)^2} \end{aligned}$$

Example : Differentiate $y = \frac{\ln x}{e^{2x}}$

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{e^{2x}\frac{1}{x} - \ln x.2e^{2x}}{e^{4x}} \\ &= \frac{e^{2x}\left(\frac{1}{x} - 2\ln x\right)}{e^{4x}} \\ &= \frac{\frac{1}{x} - 2\ln x}{e^{2x}} \end{aligned}$$

Exercise 4: Differentiate

1. $f(x) = \sin x \cos x$
2. $y = \cos\theta\left(1 - \sin\theta\right)^{-1}$
3. $g(\theta) = e^{\theta}\left(\tan\theta - \theta\right)$
4. $y = \sqrt{x}\sin x$
5. $y = te^{t}\csc t$

Exercise 5: Differentiate

1. $y = 2^{\sin \pi x}$
2. $f(x) = \sin \sin \sin x$
3. $y = \sqrt{1 + \sqrt{1 + x}}$
4. $g(x) = \cos \left(\frac{1+e^{2x}}{1-e^{2x}} \right)$
5. $q = 2^{3^{t^2}}$

Exercise 6 :

1. Find $h''(r)$ if $h(r) = \ln\left(\pi r^3\right)$
2. Find $f''(2)$ if $f(t) = e^t t^e$
3. Find $g^{(100)}(x)$ if $g(x) = \sin 2x$

Outline

Theorem

The Mean Value Theorem

1. *f is continuous on the closed interval [a,b].*
2. *f is differentiable on the open interval (a,b).*

Then there is a number c in (a,b) such that,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or, equivalently,

$$f(b) - f(a) = f'(c)\,(b - a)$$

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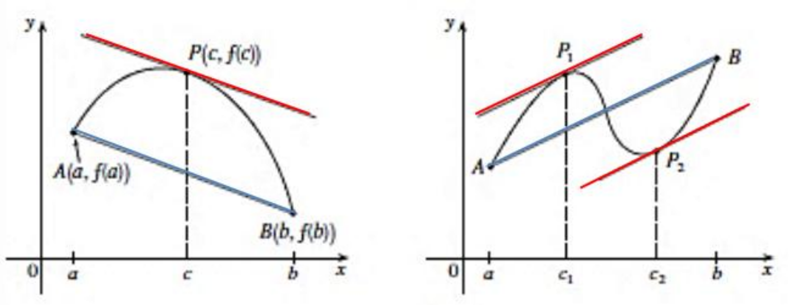

To illustrate the Mean Value Theorem with a specific function, let's consider

$$f(x) = x^3 - x, a = 0, b = 2.$$

Since f is a polynomial, it is continuous and differentiable for all x, so it is certainly continuous on [0, 2] and differentiable on (0, 2).

Therefore, by the MVT, there is a number $c \in (0, 2)$ such that

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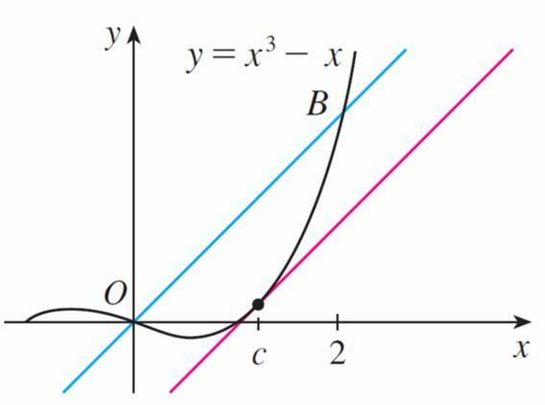

The tangent line at this value of *c* is parallel to the secant line OB.

Theorem

If $f'(x) = 0$ for all x in an interval (a, b), then f is constant on (a, b).

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Let x_1 and x_2 be any two numbers in (a, b) with $x_1 < x_2$.

Since f is differentiable on (a, b), it must be differentiable on (x_1, x_2) and continuous on $[x_1, x_2]$.

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Outline

- **Indeterminate form of type 0/0**

Limit of the form

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

in which $f(x) \longrightarrow 0$ and $g(x) \longrightarrow 0$ as $x \longrightarrow a$

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Suppose that f and g are differentiable functions on an open interval containing $x = a$, except possible at $x = a$, and that

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E.g. Find the limit

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

Using L'Hôpital's rule, and check the result by factoring.

Sol:

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \frac{2^2 - 4}{2 - 2} \quad 0/0 \textit{ form}$$

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$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{\frac{d}{dx}\left(x^2 - 4\right)}{\frac{d}{dx}(x - 2)}$$

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(1)

$$\lim_{x\to 0}\frac{sin2x}{x}=\frac{sin0}{0}=\frac{0}{0}\ form$$

Applying L'Hôpital's rule

$$\lim_{x\to 0}\frac{sin2x}{x}=\lim_{x\to 0}\frac{\frac{d}{dx}(sin2x)}{\frac{d}{dx}(x)}$$

$$=\lim_{x\to 0}\frac{2cos2x}{1}$$

$$=2cos(0)=2$$

Sol:

(1)

$$\lim_{x \to 0} \frac{sin2x}{x} = \frac{sin0}{0} = \frac{0}{0} \; form$$

Applying L'Hôpital's rule

$$\lim_{x \to 0} \frac{sin2x}{x} = \lim_{x \to 0} \frac{\frac{d}{dx}(sin2x)}{\frac{d}{dx}(x)}$$
$$= \lim_{x \to 0} \frac{2cos2x}{1}$$
$$= 2cos(0) = 2$$

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$$\lim_{x\to\pi/2}\frac{1-sinx}{cosx}=\frac{1-sin\frac{\pi}{2}}{cos\pi/2}=\frac{1-1}{0}=\frac{0}{0}\ form$$

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$$\lim_{x\to\pi/2}\frac{1-sinx}{cosx}=\lim_{x\to\pi/2}\frac{\frac{d}{dx}(1-sinx)}{\frac{d}{dx}(cosx)}$$

$$=\lim_{x\to\pi/2}\frac{-cosx}{-sinx}$$

$$=\frac{cos\pi/2}{sin\pi/2}$$

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$$\lim_{x\to 0}\frac{e^x-1}{x^3}=\frac{e^0-1}{0}=\frac{1-1}{0}=\frac{0}{0}\ \textit{form}$$

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$$\lim_{x\to 0}\frac{e^x-1}{x^3}=\lim_{x\to 0}\frac{\frac{d}{dx}(e^x-1)}{\frac{d}{dx}(x^3)}$$
$$=\lim_{x\to 0}\frac{e^x}{3x^2}$$
$$=+\infty$$

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- **Indeterminate form of type ∞/∞**

The Limit of a ratio, $\frac{f(x)}{g(x)}$ in which the numerator has limit ∞ and the denominator has the limit ∞ is called an indeterminate form of type ∞/∞

- **L'Hôpital's Rule for ∞/∞**

Suppose f and g are differentiable functions on an open interval containing $x = a$, except possibly at, $x = a$ and that

$$\lim_{x \to a} f(x) = \infty \quad \text{and} \quad \lim_{x \to a} g(x) = \infty$$

If $\lim_{x \to a} \left[\frac{f'(x)}{g'(x)} \right]$ exists, or if this limit is $+\infty$ or $-\infty$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

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E.g. In each part confirm that the limit is an indeterminate form of type ∞/∞ and evaluate it using L'HÔPITAL's rule.

1. $\lim_{x \to +\infty} \frac{x}{e^x}$
2. $\lim_{x \to 0^+} \frac{ln(x)}{csc(x)}$

Sol:

(1)

$$\lim_{x \to +\infty} \frac{x}{e^x} = \frac{\infty}{e^\infty} = \infty/\infty \ form$$

Applying L'Hôpital's rule

$$\lim_{x \to +\infty} \frac{x}{e^x} = \lim_{x \to +\infty} \frac{\frac{d}{dx}(x)}{\frac{d}{dx}(e^x)}$$

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Applying L'Hôpital's rule

$$\begin{aligned}\lim_{x\to+\infty} \frac{x}{e^x} &= \lim_{x\to+\infty} \frac{\frac{d}{dx}(x)}{\frac{d}{dx}(e^x)} \\ &= \lim_{x\to+\infty} \frac{1}{e^x} \\ &= 0\end{aligned}$$

(2)

$$\lim_{x\to 0^+} \frac{ln(x)}{csc(x)} = \frac{ln(0)}{csc(0)} = \infty/\infty \ form$$

Applying L'Hôpital's rule

$$\lim_{x\to 0^+} \frac{ln(x)}{csc(x)} = \lim_{x\to 0^+} \frac{\frac{d}{dx}(ln(x))}{\frac{d}{dx}(csc(x))}$$
$$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{-csc(x)cot(x)}$$
$$= \infty/\infty \ form$$

Any additional application of L'Hôpital's rule will yield powers of $\frac{1}{x}$ in the numerator and expressions involving $csc(x)$ and $cot(x)$ in the denominator.

(2)

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$$\lim_{x\to 0^+} \frac{ln(x)}{csc(x)} = \lim_{x\to 0^+} \frac{\frac{d}{dx}(ln(x))}{\frac{d}{dx}(csc(x))}$$
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Any additional application of L'Hôpital's rule will yield powers of $\frac{1}{x}$ in the numerator and expressions involving $csc(x)$ and $cot(x)$ in the denominator.

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$$\lim_{x\to 0^+}\left(-\frac{sinx}{x}tanx\right) = -\lim_{x\to 0^+}\frac{sinx}{x}\lim_{x\to 0^+} tanx$$
$$= -(1)(0) = 0$$

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The limit of an expression that has one of the forms

$$\frac{f(x)}{g(x)}, f(x)\cdot g(x), f(x)^{g(x)}, f(x)-g(x), f(x)+g(x)$$

is called and indeterminate form if the limits $f(x)$ and $g(x)$ individually exert conflicting influences on the limit of the entire expression.

- **Indeterminate form of type** $0\cdot\infty$

For example

$\lim_{x\to 0^+} xln(x) = 0\cdot\infty$ Indeterminate form

On the other hand

$\lim_{x\to+\infty}\sqrt{x}\left(1-x^2\right) = +\infty(-\infty) = -\infty$ Not an indeterminate form

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Indeterminate form of type $0 \cdot \infty$ can sometimes be evaluated by rewriting the product as a ratio, and then applying L'Hôpital's rule for indeterminate form of type 0/0 or ∞/∞.

E.g. Evaluate

1. $lim_{x\to 0^+} xln(x)$
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Sol:

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$$\lim_{x\to\pi/4}(1-tanx)(sec2x) = \lim_{x\to\pi/4}\frac{1-tanx}{cos2x} = \lim_{x\to\pi/4}\frac{\frac{d}{dx}(1-tanx)}{\frac{d}{dx}(cos2x)}$$

$$= \lim_{x\to\pi/4}\frac{-sec^2x}{-2sin2x}$$

$$= \frac{(sec\frac{\pi}{4})^2}{2sin(\frac{2\pi}{4})} = \frac{2}{2} = 1$$

(2)

$$\lim_{x\to\pi/4}(1 - tanx)(sec2x) = 0 \cdot \infty$$

Rewriting

$$\lim_{x\to\pi/4}(1 - tanx)(sec2x) = \lim_{x\to\pi/4} \frac{1 - tanx}{\frac{1}{sec2x}}$$
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- **Indeterminate form of type $\infty - \infty$**

A limit problem that leads to one of the expressions

1. $(+\infty) - (+\infty)$
2. $(-\infty) - (-\infty)$
3. $(+\infty) + (-\infty)$
4. $(-\infty) + (+\infty)$

is called an indeterminate form type $\infty - \infty$

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3. $(-\infty) + (-\infty)$
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are not indeterminate, since two terms work together.

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E.g. Evaluate

$$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{sinx}\right)$$

Sol:

$$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{sinx}\right)=\left(\frac{1}{0}-\frac{1}{sin0}\right)=\infty-\infty \textit{ form}$$

Rewriting

$$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{sinx}\right)=\lim_{x\to 0^+}\left(\frac{sinx-x}{xsinx}\right)=\left(\frac{sin0-0}{0sin0}\right)=0/0 \textit{ form}$$

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$$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{sinx}\right)$$

Sol:

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$$\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{sinx} \right)$$

Sol:

$$\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{sinx} \right) = \left(\frac{1}{0} - \frac{1}{sin0} \right) = \infty - \infty \textit{ form}$$

Rewriting

$$\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{sinx} \right) = \lim_{x \to 0^+} \left(\frac{sinx - x}{xsinx} \right) = \left(\frac{sin0 - 0}{0sin0} \right) = 0/0 \textit{ form}$$

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Again Applying L'Hôpital's rule

$$\begin{aligned}\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{sinx}\right) &= \lim_{x\to 0^+}\left(\frac{\frac{d}{dx}(cosx-1)}{\frac{d}{dx}(sinx+xcosx)}\right)\\ &= \lim_{x\to 0^+}\left(\frac{-sinx}{cosx-xsinx+cosx}\right)\\ &= \frac{-sin0}{cos0-0sin0+cos0}\\ &= \frac{0}{1-0+1}\\ &= 0\end{aligned}$$

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$$\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{sinx} \right) = \lim_{x \to 0^+} \left(\frac{\frac{d}{dx}\,(cosx - 1)}{\frac{d}{dx}\,(sinx + xcosx)} \right)$$

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- **Indeterminate forms of type** $0^0, \infty^0, 1^\infty$

Limits of the form

$$\lim f(x)g^{(x)}$$

can give rise to indeterminate forms of the types $0^0, \infty^0$ *and* 1^∞

E.g.

$$\lim_{x\to 0^+}(1+x)^{\frac{1}{x}} \quad (1^\infty)\ form$$

pause It is indeterminate because the expressions $1+x$ and $\frac{1}{x}$ gives 1 and ∞ respectively. Two conflicting influences. Such inderminate form can be evaluated by first introducing a dependent variable

$$y = f(x)^{g(x)}$$

$$ln(y) = ln(f(x)^{g(x)})$$

$$= g(x).ln(f(x))$$

The limit of ln(y) will be an indeterminate form of type $0 \cdot \infty$

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E.g.

$$\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e \quad Note:\ a^x = e^x ln(a)$$

Sol: Let $y = (1+x)^{\frac{1}{x}}$

$$ln(y) = ln(1+x)^{\frac{1}{x}} \Rightarrow ln(y) = \frac{1}{x} ln(1+x)$$

$$\lim_{x\to 0} lny = \lim_{x\to 0} \frac{ln(1+x)}{x} = \frac{ln(1+0)}{0} \quad (0/0\ form)$$

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Sol: Let $y = (1+x)^{\frac{1}{x}}$

$$ln(y) = ln(1+x)^{\frac{1}{x}} \Rightarrow ln(y) = \frac{1}{x} ln(1+x)$$

$$\lim_{x\to 0} lny = \lim_{x\to 0} \frac{ln(1+x)}{x} = \frac{ln(1+0)}{0} \quad (0/0 \; form)$$

Applying L'Hôpital's rule,

$$\lim_{x \to 0} lny = \lim_{x \to 0} \frac{\frac{1}{1+x}}{1}$$

$$= \lim_{x \to 0} \frac{1}{1+x} = 1$$

$$ln(y) \to 1 \ as \ x \to 0$$

$$\Rightarrow e^{ln(y)} \to e^{1} \ as \ x \to 0$$

$$\Rightarrow y \to e \ as \ x \to 0$$

Thus

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End!