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Modeling of an Optimal Outbound Logistics System (A Contemporary Review Study on effects of Vehicle Routing, Facility Location and Locational Routing Problems)

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ABSTRACT*: In recent years, problems related to warehouse management has become an important consent in the field of logistics management research and has been widely used in analyzing through transportation and logistics distribution systems. This paper aims to conduct a comprehensive literature review on recent model developments and improvements involving Vehicle Routing Problem(VRP), Facility Location Problems and Location Routing Problems along with its variants. VRP is a widely studied combinatorial optimization problem in operational research and computer science. The VRP is categorized by Capacitated Vehicle Routing Problem (CVRP), Vehicle Routing Problem with Time Windows (VRPTW), Vehicle Routing Problem with Multi-Depot (MDVRP) and their variants. Further discussions has been carried on in the areas of classification of VRP, summarization of the common constraints of VRP, model algorithm developed in recent years. Finally, it is being analyzed, the future model implications of VRP, and it is considered that the Intelligent Vehicle Routing Problem and Intelligent Heuristic Algorithm will be an important field of future research.The review study also aims at finding the effect and developments in Facility Location Problems which are classical optimization problems having an extensive application in transportation, distribution, production and supply chain management decisions.This will also provide a broader overview of Location Routing Problems, indicating how they are formulated and what are the proposed solutions.*

Key words: *Capacitated Vehicle Routing Problem, Exact Methods,Heuristics, Location Routing Problem, Meta – Heuristics, Vehicle Routing Problem, Vehicle Routing Problem with Time Windows, Vehicle Routing Problem with Multi-Depot Optimization.*

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I. INTRODUCTION TO VEHICLE ROUTING PROBLEMS

The VRP is used in Logistics and supply chain management in physical delivery of goods and services. The VRP can be simply defined as the problem of designing least cost delivery routes from a depot to a set of geographically scattered customers, subject to the side constraints. This is central to distribution management and must be routinely solved by carriers. There are several variants to the VRP and are formulated based on the nature of the goods transporting, the service quality and the characteristics of customers and vehicles. Dantzig &Ramser (1959) initially introduced ``Truck Dispatching Problem'', modeling a fleet of homogeneous trucks to serve the demand for oil of a number of gas stations from a central hub and with a minimum travell distance.Clarke & Wright (1964) rerarranged this problem to a linear optimization problem that is commonly encountered in the domain of logistics and transport: i.e., how to serve a set of customers, geographically dispersed around the central depot, using a fleet of trucks with varying capacities becoming VRP, one of the most widely used phenominasin the field of Operations Research.

Present VRP models, are massively different from the model introduced by Dantzig &Ramser (1959) and Clarke & Wright (1964), as it gradually aims to incorporate real-life complexities, for instance dependent travel times (reflecting traffic congestion), time windows for pickup and delivery, and input information (e.g., demand information) that changes over time. Accoding to Lenstra&Rinnooy Kan (1981), the VRP is an NP-hard problem. Which explains the exact algorithms only efficient for the instances of having in small scale. Heuristics and metaheuristics are often more suitable for more practical implications, in view ofreal life problems that are considerably larger in scale (e.g., a company may need to supply thousands of customers from dozens of depots with numerous vehicles and subjects to a variety of constraints).

Further, VRP offers a varied range of heuristics and meta – heuristics approaches, that are introduced in Laporte (2009),Gendreau et al. (2002) and Cordeau et al. (2005). The VRP is so widely studied because of its wide applicability and its importance in determining efficient strategies for reducing transportation cost in distribution networks. Consequently, current research concentrates on approximating algorithms whichare capable of finding high quality solutions in a limited time frame, in order to be applicable to real life problem scenarios that are characterized by large vehicle fleets and affected significantly to logistics and distribution strategies.

The development of several types of VRPs were found along with methods for determining the shortest route. Goetschalckx(2011) defines VRP as a problem of determining the shortest route from a vehicle that starts from one depot to nmultiple destinations to meet customer needs. Where the vehicle has a certain capacity, every vehicle starts from the depot and returns to the depot. Each customer can only be visited once. The purpose of VRP is to meet the needs of every customer with minimal cost. The cost is directly propotional to the total distance travelled by all vehicles , thus the VRP determines the shortest distance.

Currently VRP software is being used by manypublic, private and multinational companies, and in large variety of industry sectors amongst them, Coca-Cola Enterprises and Anheuser-Busch Inbev are most significant. (Drexl, 2012; Partyka& Hall, 2014). The VRP literature has been growing exponentially at a rate of 6% every year. This popularity makes it difficult to keep track of the developments in the field, and to have a clear overview of which variants and solution methods are relatively novel.

The VRP generalizes the well-known Traveling Salesman Problem(TSP) but is much more difficult to solve in practice. Whereas there exist exact algorithms capable of routinely solving TSPs containing a huge number of vertices (Applegate et al. 2007). This is not the circumstance of the VRP for which the best exact algorithms can only solve instances involving approximately 100 vertices (Fukasawa et al. 2006; Baldacci et al. 2008). Because of the real incidents of the VRP often exceeds this size and solutions must often be determined quickly, most commonly used algorithms are heuristics. In recent years, several powerful metaheuristics algorythmshave further been developed.

Importantly the following are some of the constraints or limitations that must be met in the VRP.

a) The vehicle route starts from the depot and ends at the depot.

b) Each customer must be visited once with one vehicle.

c) Vehicles used are homogeneous with a certain capacity so that consumer demand
on each route traversed should not exceed the capacity of the vehicle.

d) If the vehicle capacity has reached the limit, then the next consumer will be served by the next shift.

1.1 Capaciated vehicle routing problem and its variants

The Capacitated Vehicle Routing Problem (CVRP) is one of the fundamental problems in the combinatorial optimization with a number of practical applications in transportation, distribution and logistics and fulfilling an extension of VRP. The aim of CVRP is to find a set of minimum total cost routes for a fleet of vehicles with a significant capacity based at a single depot, to serve a set of customers under the following constraints:

(1) each route begins and ends at the depot,

(2) each customer is visited exactly once,

(3) the total demand of each route does not exceed the capacity of the vehicle Laporte (2007).

The first mathematical formulation and algorithm for the solution of the CVRP was proposed by Dantzig &Ramser(1959) and Clarke & Wright (1964) proposed the first heuristic for this problem. Presently, many solution methods for the CVRP have been published. General surveys can be found in Toth & Vigo (2002) and Laporte (2007). The CVRP falls in to the category of NP hard problems that can be exactly solved only for small scaled instances of a problem. Therefore, many researchers have concentrated on developing heuristic algorithms to solve these problems (Gendreau& Potvin (2010),Laporte&Ropke(2014). For a long term, numerous studies have been focused on solving these problems and several approaches have been extended from direct tree search with Branch-and-Bound (Christofides&Eilon (1969) to column generation, BranchandCut algorithms and metaheuristics.Heceforth, all of these methods have provided a better solution based on CVRP problems.

In some versions of the CVRP one also has to obey a route duration constraint that limits the lengths of the feasible routes. The VRPTW extends the CVRP by associating time windows to the customers. A time window could be definedas an interval during which the customer must be visited. The OVRP is closely related to the CVRP, but contrary to the CVRP a route ends as soon as the last customer has been served as the vehicles do not need to return to the depot. The MDVRP extends the CVRP by allowing multiple depots.

1.1.1 Mathematical formulation ofthe CVRP

Let G = (V,H) be a complete directed graph with V = {0,1,2,...,n} as the set of nodes and H = {(i,j) : i,j∈V,i≠j} as the set of edges, where node 0 represents the depot for a fleet of k vehicles with the same capacity Q and remaining n nodes represent geographically spread customers. Each customer i∈ V −{0} has a certain positive demand $d_i \leq Q$. The non negative travel cost $c_{ij}$is associated with each edge (i,j) ∈ H. The cost matrix is symmetric, i.e. $c_{ij}= c_{ji}$for all i,j∈V,i≠j and satisfies the triangular inequality, $c_{ij}+c_{jk}\geq c_{ik}$for all i,j,k∈ V , Toth & Vigo (2002).The minimum number of vehicles needed to serve all customers is $[\sum_{i=1}^{n} d_i]/Q]$

1.1.2 Two – index vehicle flow formulation

In the twoindex formulation, the binary decision variable is defined as x_{ij}that assumes value 1 if and only if there is a route that goes from customer ito j directly, for i,j∈V. In addition, $y_j$is a continuous decision variable corresponding to the cumulated demand on the route that visits node j ∈Vup to this visit. With these parameters and decision variables,

The two – index flow formalulation of the CVRPMunari et.al (2017) cangiven by :

$$\text{Min} \sum_{i=0}^{n+1} \sum_{j=0}^{n+1} (c_{ij}\, x_{ij}) - [1]$$

Such that

$$\sum_{j=1, j\neq i}^{n+1} (x_{ij}) = 1, \quad i \in \{1,2,3 \ldots \ldots . . n\} - - - - - [2]$$

$$\sum_{i=0, i\neq h}^{n} (x_{ih}) - \sum_{j=1, j\neq h}^{n+1} (x_{hj}) = 0, \quad h \in \{1,2,3 \ldots \ldots . . n\} - [3]$$

$$\sum_{j=1}^{n} (x_{oj}) \leq K, - - - - - - - - - - - - - - - [4]$$

$$y_j \geq y_i + q_j x_{ij} - Q\,(1- x_{ij}), \; i,j \in \{0,1 \ldots . . , n+1\} \quad [5]$$

$$d_i \leq y_i \leq Q\,(1- x_{ij}), \; i \in \{0,1 \ldots \ldots . . , n+1\} - - - \quad [6]$$

$$x_{ij} \in \{0,1\}, i,j \in \{0, \ldots \ldots . . n, n+1\} - - - - - - - - - - - - - - \quad [7]$$

Constraint (2) ensure that all customers are visited exactly once. Constraint (3) guarantees that the correct flow of vehicles through the edges, by stating that if a vehicle arrives to a node h ∈V, then it must depart from this node. Constraint (4) limits the maximum number of routes to K, the number of vehicles. Constraints (5) and (6) together ensures that the vehicle capacity is not exceeded. The objective function is depicted by (1) and imposes that the total travel cost of the routes is minimized.

Constraint (5) also avoid subtours in the solution, i.e. cycling routes that do not pass through the depot. Different types of constraints are proposed in the literature to impose vehicle capacities and/or avoid subtoursIrnich et al. 2014. The advantage of using constraint (5) and (6) is that the model has a polynomial number of constraints in terms of the number of customers. However, the lower bound provided by the linear relaxation persist to capacity constraints that results in better lower bounds, even though the number of constraints becomes exponential in terms of the number of customers, requiring the use of a branch-and-cut strategy Semet et al. 2014.

1.1.3 Three – index vehicle flow formulation

The binary decision variable x_{rij} is defined to indicate if the vehicle r , r ∈ $\{1,2,3 \dots \dots.. p\}$ tranverses an edge (i,j) in an optimal solution. The integer linear programming model of the CVRP Janacek et.al (2013) can givenby :

$$\text{Min} \sum_{r=1}^{p} \sum_{i=0}^{n} \sum_{j=0, i \neq j}^{n} c_{ij} \left(x_{rij}\right) ------------------ [1]$$

Subject to

$$\sum_{r=1}^{p} \sum_{i=0, i \neq j}^{n} \left(x_{rij}\right) = 1 \qquad \forall\, j \in \{1,2,3 \dots \dots.. n\} - ------- [2]$$

$$\sum_{j=1}^{n} \left(x_{roj}\right) = 1 \qquad \forall\, r \in \{1,2,3 \dots \dots.. p\} ----- [3]$$

$$\sum_{i=0, i \neq j}^{n} \left(x_{rij}\right) = \sum_{j=1}^{n} \left(x_{rji}\right) \quad \forall\, j \in = \{0,1,2 \dots \dots.. n\}\ , r \in \{1, \dots \dots.. p\} - [4]$$

$$\sum_{i=0}^{n} \sum_{j=1, i \neq j}^{n} d_j \left(x_{rij}\right) \leq Q \qquad \forall\, r \in \{1,2,3 \dots \dots.. p\} - [5]$$

$$\sum_{r=1}^{p} \sum_{i=0}^{n} \sum_{j \in S, i \neq j}^{n} x_{rij} \leq |S| - 1 \ \ \forall\, S \dot{\subseteq} \{1,2,3 \dots \dots.. n\} – [6]$$

$$x_{rij} \in \{0, 1\}, \qquad \forall\, r \in = \{1,2 \dots \dots.. p\}\ , i, j \in \{0,1, n\}, i \neq j -- - [7]$$

The objective function (1) minimizes the total transportation cost. The model constraint (2) are the degree constraint and ensure that each customer is visited by exactly one vehicle. The flow constraints (3) and (4) assurance that each vehicle can leave the depot only once, and the number of the vehicles arriving at every customer and entering the depot is equal to the number of the vehicles leaving. In the constraints (5) the capacity constraints are stated, making sure that the sum of the demands of the customers visited in a route is less than or equal to the capacity of the vehicle performing the service. The sub-tour elimination constraint (6) guarantee that the solution contains no cycles disconnected from the depot. The remaining obligatory constraint (7) specify the definition domains of the variables.The number of inequalities of the sub-tour elimination constraints grows exponentially with the number of nodes.

1.2 Capacitated Vehicle Routing Problem with Time Windows (CVRPTW)

The Capacitated Vehicle Routing Problem with Time Windows is the CVRP with additional time window constraints. The objective of the VRPTW is to serve a number of customers within predefined time windows at minimum cost (interms of distance travelled), without violating the capacity and total tour time constraints for each vehicle. In real world, some customers can only be served in a given time period, we refer this time period as the time windows. Consequently, time window constraints should be taken into consideration. In CVRPTW, customers could begin to be served within a time window $[E_i,L_i]$. If vehicles arrive at customer ibefore E_i, vehicles could wait until E_i, since there is no additional cost to wait. If the vehicles arrive at customer iafter L_i, as the customer is not available, the vehicle cannot pick up anything and additional penalty costs should be taken into consideration. Solomon(1987) proposed the sequential insertion heuristic to solve CVRPTW. Accordingy, a problem set with different percentages of time windows, positioning and tightness were generated to test performance of algorithms used in CVRPTW.

1.2.1 Mathematical formulation of the CVRPTW

The constraints of the Vehicle Routing Problem with Time Windows (VRPTW) consist of a set of identical vehicles, a central depot node, a set of customer nodes and a network connecting the depot and customers. There are N + 1 customers and K vehicles. The depot node is denoted as customer 0. Each edge in the network represents a connection between two nodes and also specifies the direction it travels. Each route starts from the depot. The number of routes in the network is equal to the number of vehicles used. One vehicle is dedicated to one route. A cost $\mathbf{c_{ij}}$ and a travel time $\mathbf{t_{ij}}$are associated with each edge of the network.

In Solomon's 56 CVRPTW "100" Customers instancesSolomon (1987), all distances are represented by Euclidean distance, and the speed of all vehicles is assumed to be unity. That is, it takes one unit of time to travel one unit of distance. This assumption makes the problem simpler, because numerically the travel cost $c_{ij,}$ the travel time t_{ij} and the Euclid- ean distance between the customer nodes equal each other.

Each customer in the node can be visited only once by one of the vehicles. Every vehicle has the same capacity q_k and every customer has a varying demand m_i. q_k must be greater than or equal to the sum of all demands on the route travelled by the vehicle k, which means that no vehicle can be overloaded. The time window constraint is represented by a predefined time interval, given an earliest arrival time and latest arrival time. The vehicles must arrive at the customers not later than the latest arrival time. If vehicles arrive earlier than the earliest arrival time, waiting occurs. Each customer also imposes a service time to the route, taking into consideration the loading/unloading time of goods. In Solomon's instances, the service time is assumed to be unique regardless of the load quantity needed to be handled. Vehicles are also supposed to complete their individual routes within a total route time, which is essentially the time window of the depot.

There are three types of principal decision variables in VRPTW. The principal decision variable x_{ijk} (i, j∈{0, 1, 2, ···, n}; k∈ {1, 2, ···, K}; i≠ j) is 1 if vehicle k travels from customer i to customer j, and 0 otherwise. The decision variable T_i denotes the time when a vehicle arrives at the customer, and w_i denotes the waiting time at nodei. The objective is to design a network that satisfies all constraints, at the same time minimizing the total transportation cost. The integer liner programming model of the the VRPTW Kumar &Pannerselvam (2012)can givenby :

Decision Variables

T_i= arrival time at node i
w_i= wait time at node i
$x_{ijk}\in \{0,1\}$, 0 if there is no edge from node ito node j, and 1 otherwise,
$i\neq j\,; i,j \in \{0,1,2,\ldots\ldots.n\}$

Parameters:

K = total number of vehicles
n = total number of customers
c_{ij} =cost incurred on edge from node ito j
t_{ij} = travel time between node iand j
m_i =demand at node i
q_k =capacity of vehicle k
e_i =earliest arrival time at node i
l_i = latest arrival time at node i
f_i = service time at node i
r_k =maximum route time allowed for vehicle k

$$Min \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{j \neq i, k=1}^{K} c_{ij} \left(x_{ijk} \right)$$

Subject to :

$$\sum_{k=1}^{K} \sum_{j=1, i \neq j}^{n} \left(x_{ijk} \right) \leq K \quad , \quad for\ i = 0 -------[1]$$

$$\sum_{j=1}^{n} \left(x_{ijk} \right) = 1\ , \quad for\ i = 0\ and\ k\ \in (1,2, \ldots..K) ----[2]$$

$$\sum_{j=1}^{n} \left(x_{jik} \right) = 1\ , \quad for\ i = 0\ and\ k\ \in (1,2, \ldots..K) ----[3]$$

$$\sum_{k=1}^{K} \sum_{j=0, j \neq i}^{n} \left(x_{ijk} \right) = 1 \quad , for\ i\ \in (1,2, \ldots..n) ------[4]$$

$$\sum_{k=1}^{K} \sum_{i=0, i \neq j}^{n} \left(x_{ijk} \right) = 1 \quad , for\ j\ \in (1,2, \ldots..n) -----------[5]$$

$$\sum_{i=1}^{n} m_i \sum_{j=0, j \neq i}^{n} \left(x_{ijk} \right) \leq q_k \quad , for\ k\ \in (1,2, \ldots..K) ---------[6]$$

$$\sum_{i=0}^{n} \sum_{j=0, j \neq i}^{n} \left(x_{ijk} \right) (t_{ij} + f_i + w_i) \leq r_k \quad , for\ k\ \in (1,2, \ldots..K) --- [7]$$

$T_0 = w_0 = f_0 = 0$ $-----------------------[8]$

$$\sum_{k=1}^{K} \sum_{i=0, i \neq j}^{n} \left(x_{ijk} \right) (T_i + t_{ij} + f_i + w_i) \leq T_j \quad , for\ i\ \in (1,2, \ldots..K) -- [9]$$

$$(e_i) \leq (T_i + w_i) \leq T_j\ for\ i\ \in (1,2, \ldots..n) ------------ [10]$$

The objective function minimizes the total cost of travel of all the vehicles in completing their tours. Constraint set [1] guarantees that the number of tours is K by selecting at most K outgoing edges from the depot (i = 0).The constraint set [2] ensures that for each vehicle, there is exactly one outgoing edge from the depot is selected. Similarly, the constraint set [3]ensures that for each vehicle, there is exactly one edgeentering into the node with respect to depot (i = 0). These two constraint sets [2] , [3] jointly ensure that a complete tour for each vehicle is ensured.The constraint set [4] makes sure that from each node i only one edge for each vehicle originates from it. The constraint set [5] ensures that for each node j, only one edge for each vehicle enters into it. These two constrains set [4] and [5] make sure that each vehicle visits each node only one time.The constraint set [6] sees that for each vehicle, the total demandallocated to it is less than or equal to its capacity. The constraint set [7] confirms that the total time of travel of the route of each vehicle is less than or equal to the maximum route time allocation to that vehicle.

The constraint set [8] shows the arrival time, waiting time and service time of each vehicle at the depot to be zero. The constraint set [9] assurances that the arrival time of each vehicle at the node j is less than the specified arrival time at that node. The constraint set [10] guarantees that the sum of the arrival time and the waiting time of each vehicle at each node i is more than equal to the initial arrival time at that node and less than or equal to the latest arrival time at that node i, i = 1, 2, 3, ⋯, n. Constraint sets [8],[9] and [10] define the time windows. These formulations completely specify the feasible solutions for the VRPTW.

A constraint is called hard if it must be satisfied, while it is called soft if it can be violated. The violation of soft constraints is usually penalized and added to the objective function.

1.3 Vehicle Routing Problem with Multi-Depot (MDVRP)

The Multi-Depot Vehicle Routing Problem (MDVRP) is a generalization of classical Vehicle Routing Problem (VRP).The objective of this problem is to find the routes for vehicles to service all the customers at a minimal cost which is in terms of number of routes and total travel distance without violating the capacity and travel time constraints of the vehicles. MDVRP is a NP-hard problem which is more advantageous than VRP which simultaneously determines the routes for several vehicles from multiple depots to a set of customers and then return to the same depot. Further Purpose of the MDVRP is to minimize the total delivery distance or time spent in serving all consumers. Lesser the delivery time, higher the customer satisfaction. Consequesntly, fewer vehicles also can reduce the total cost of operation, thus the objective can also be minimizing the number of vehicles. Though there may be several objectives, the aim of MDVRP is to increase the efficiency of delivery.

The Multi-Depot Vehicle Routing Problem (MDVRP) Carlssony et .al (2010)is a generalization of the Single-Depot Vehicle Routing Problem (SDVRP) in which vehicle(s) start from multiple depots and return to their depots of origin at the end of their assigned tours. The traditional objective in MDVRP is to minimize the sum of all tour lengths, and existing literature handles this problem with a variety of assumptions and constraints. Multi Depot Vehicle Routing Problem (MDVRP) implicates number of depots. In the traditional approach it was considered that each vehicle is assigned same number of nodes, yet in MDVRP same number of vehicles is assigned to each depot. In the former case i.e in VRP the results were poor thus this technique is accepted as this gives better results than VRP. In most of the practical VRPs, demands at the customer nodes vary due to various factors: being location and temporal seasonal factors. A network routing topology generated by solving min-max MDVRP results in a set of "daisy-chain network" configurations that minimize the maximum latency between a server and the client. This can be advantageous in situations in which the server-client connection cost is high but the client-client connection cost is low.Vehicles should start from the depot and then return back to the depot after serving an extensive amount of customers. Every customer has a demand which variesin stochastics. Vehicles are assigned to the customers and one customer is served by only one vehicle.

The MDVRP consists of building a set of vehicle routes in such a way that: (1) each route starts and ends at the same depot, (2) each customer is visited exactly once by a vehicle, (3) the total demand of each route does not exceed the vehicle capacity, (4) the total duration of each route (including travel and service times) does not exceed a preset limit so that (5) the total routing cost is minimized.

For the MDVRP, there are several models developed (exact and approximate approaches). Due to its NP-hard combinatorial nature, the models proposed in the literature are mostly heuristics-based. There are still few exact algorithms in the literature. Laporte et al. (1984), as well as Laporte et al.(1988), developed exact branch and bound algorithms for solving the symmetric and asymmetric version of the MDVRP, respectively. Baldacci &Mingozzi (2009) developed an exact method for solving the Heterogeneous Vehicle Routing Problem (HVRP) that is capable to solve, the MDVRP. This algorithm is based on the set partitioning formulation, where a procedure is applied to generate routes and three bounding procedures are used to reduce the number of variables in the formulation. However, when analysing heuristic algorithms to solve MDVRP several ones have been proposed Tillman & Cain (1972), Golden et al. (1977), Renaud et al.(1996), Salhi& Sari (1997), Lim & Wang (2005), Crevier et al. (2007).

Therefore it can be concluded that few exact models for the multi-depot problems have been proposed, while many heuristic procedures exist for the same problem. The combination of these two methods has also received little attention from the academia. Therefore, this work explores this opportunity and proposes a hybrid method combiningexplicit formulation with heuristic procedures to solve the multi-product with multi-depot vehicle routing problem.

1.3.1 Mathematical formulation of the MDVRP

The MDVRP is formulated with the objective of forming a sequence of customers on each vehicle route. The time required to travel between customers along with the depot and demands are known in advance. It is assumed that all vehicles have the same capacity, and each vehicle starts its travel from a depot, upon completion of service to customers, it has to return to the depot.
The notations used and the mathematical model are as follows :

Sets :I = Set of all depots , J = Set of all Customers , K = Set of all vehicles
Indices :i = depot index , j = customer index , k = route index
Parameters : N = Number of vehicles , C_{ij} = Distance Between point i and j , where i , j ∈ $I \cup J$
V_i =Maximum throughput at depot i ,d_j = Demand of customer j
Q_k = Capacity of vehicle (route) k

Decision Variables

x_{ijk}= 1, ifiimmediatelypreceeds j on route k
0, otherwise

z_{ij}= 1, if customer j is allotted to depoii
0, otherwise

U_{lk} = auxiliaryvariable for sub-tour elimination constraints in route k

Mathematical Model

$$Min \sum_{i \in I \cup J} \sum_{j \in I \cup J} \sum_{k \in K} c_{ij} \left(x_{ijk}\right) - - - - - - - - - - - - - - [1]$$

$$\sum_{k \in K} \sum_{i \in I \cup j} \left(x_{ijk}\right) = 1 \quad , \qquad j \in J - - - - - - - - - - - - [2]$$

$$\sum_{j \in J} d_j \sum_{i \in I \cup j} \left(x_{ijk}\right) \leq Q_k \quad , k \in K - - - - - - - - - - - \; [3]$$

$$\left(U_{lk}\right) - \left(U_{jk}\right) + N x_{ijk} \leq N - 1, \quad l, j \in J, \;\; k \in K - - - - - [4]$$

$$\sum_{j \in I \cup J} \left(x_{ijk}\right) - \sum_{j \in I \cup J} \left(x_{jik}\right) = 0 \;, \quad k \in K, j \in I \cup J \; - - - [5]$$

$$\sum_{i \in I} \sum_{j \in J} \left(x_{ijk}\right) \leq 1 \quad , k \in K - - - - - - - - - - - - - - - [6]$$

$$\sum_{j \in J} d_i \left(z_{ij}\right) \leq V_i, \qquad i \in I - - - - - - - - - - - - - - - - - [7]$$

$$- z_{ij} + \sum_{u \in I \cup J} \left(x_{iuk}\right) + \left(x_{ujk}\right) \leq 1 \;, i \in I, j \in J, k \in K - - - -[8]$$

$$\left(x_{jik}\right) = \{0,1\}, i \in I, j \in J, k \in K - - - - - - - - - - - - - - [9]$$

$$\left(z_{ji}\right) = \{0,1\}, i \in I, j \in J, - - - - - - - - - - - - - - - - - - [10]$$

$$(U_{lk}) \geq 0\,, l \in J, k \in K - - - - - - - - - - - - - - - - [11]$$

The objective function [1] minimizes the total delivery distance of all the vehicles in completing their tours.Constraint set [2] guarantees that each customer is apportioned with only one route while constraint set [3] ensures the capacity limit of vehicles. Similarly, the sub-tour avoidance is imposed by constraint set [4]and flow conservation constraint given by [5]. The route to be served and the limit on the depots are given by constraint set [6],[7] respectively. [8] Constraints specify that a customer can be assigned to a depot only if there is a route from that depot going through that customer and constraint [9], [10] jointly ensure that the binary requirements on the decision variables. Constraint [10] keep auxiliary variable as positive values. Thus the MDVRP aims at minimizing the total delivery distance by satisfying the mentioned constraints .

1.4 Exact Methods

Exact algorithms to solve VRP particularly the capacitated VRP (CVRP) include the 'branch-and-bound', the 'branch-and-cut' and the 'branch-and-price' algorithms. A branch-and-cut and branch-and-price exact algorithm have being recommended by Ropke et al. (2007) for the CVRP with Two-Dimensional Loading Constraints. Gutierrez-Jarpaet al.(2010) proposed an exact algorithm for the multiple vehicle routing problem with time windows(VRPTW). They proposed an exact branch-and-price algorithm for solving the multiple VRPTW. Column generation or Dantzig-Wolfe decomposition provides a flexible framework that can accommodate complex constraints and time-dependent costs.

An exact solution method has being developed by Aziet al.(2010) for vehicle routing and scheduling problem with soft time windows (VRPSTW). They proposed a new column generationbased exact optimization approach for the vehicle routing problem with semi soft time windows (VRPSSTW). Elementary level at shortest path problem with resource constraints and late arrival penalties is answered as a subproblem. This uses the Dantzig-Wolfe decomposition method.The authorsQureshi et al.(2009) give an exact solution approach for vehicle routing and development problem with soft time windows (VRPSTW).

1.4.1 Algorithms Based on the Set Partitioning Formulation

The Set Partitioning (SP) formulation of the CVRP was originally proposed byBallinski&Quandt(1964). A binary variable is used to represent a feasible route. The Ballinski&Quandt formulation as follows.

Let R denote a set of routes in which r denotes a specific route. Let a_{ir} be a binary coefficient equal to 1 if and only if vertexi∈V\{0} belongs to route r, let c_r^* be the optimal cost of the route r, and let y_k be a binary variable equal to 1 if and only if route r is used in the optimal solution. The problem is then as given below.

A full column generation algorithm was developed byAgarwalet al.(1989) who solved instances (n) in the range 15 to 25. According Laporte (2009), it is impractical for a direct application of this formulation, because of the large number of potential routes encountered in most nontrivial instances and of the difficulty of computing the c_r^* coefficients since it requires solving an exponential number of instances of an NP-hard problem.

Alvarenga et al.(2007) suggest a genetic and set partitioning two-phase approach for the VRPTW. The VRPTW is formulated as a set partitioning problem (SP). The genetic algorithm is based on natural reproduction, selection and evolution of Darwin's theory. Ever since, genetic algorithm has been popular because it can contribute to find good solutions for complex mathematical problems, like the VRP and other NP-hard problems, in a reasonable amount of time.Dantzig &Ramser (1959) defined how the VRP may be considered as a generalization of the travelling salesman problem (TSP). They described the generalization of the TSP with multiple salesmen and called this problem the "clover problem".

The problem of routing vehicles stationed at a central facility (depot) to supply customers with known demands, in such a way as to minimize the total distance travelled, it is denoted as the vehicle routing problem and is a generalization of the multiple travelling salesman problem that has many practical applications.

Christofideset al.(1981) current tree search algorithms for the exact solution of the VRP incorporating lower bounds computed from 1) shortest spanning k-degree centre tree (k-DCT)and 2) q-routes. The final algorithms also include problem reduction and dominance tests. They present computational results for a number of problems derived from the literature. The results of these authors show that the bounds derived from the q-routes are superior to those from k-DCT and that VRPs of up to about 25 customers can be solved exactly.

Bardet al.(2002) have developed a branch-and-cut procedure for the VRPTW. It addresses the problem of finding the minimum number of vehicles required to visit a set of nodes subject to time window and capacity constraints. The fleet is homogeneous and is located at a common depot. Each node requires the same type of service. An exact method is introduced based on branch and cut. In their computations, they obtain ever increasing lower bounds on the optimal solution. This is done by solving a series of relaxed problems that incorporate newly found valid inequalities. They obtain feasible solutions or upper bounds using greedy randomized adaptive search procedure (GRASP). They also introduce a wide variety of cuts to tighten the linear programming (LP) relaxation of the original mixed-integer program. To find violated cuts, it is necessary to solve a separation problem.

1.4.2 Heuristic Approaches

Fisher&Jaikumar (1981) developed locates m seeds and constructs a cluster for each seed so as to minimize the sum of customers to seed distances, while filling the capacity constraint. This is accomplished by solving a Generalized Assignment Problem (GAP). A route is then determined on each cluster by solving a TSP. Some procedures for selecting the seeds are described in the study of Baker &Sheasby (1999). Exact comparisons with other algorithms are difficult to make because the distance rounding convention used in the experiments is not specified. It is also interesting from a methodological point of view because if can benefit from algorithmic improvements for the Generalized Assignment Problem (GAP) or for the TSP.

Gandreauet al.(2006) introduced a neighborhood search heuristics to optimize the planned routes of vehicles in a context where new requests, with a pickup and a delivery location, occur in realtime. This is a dynamic vehicle routing problem (DVRP). Within this framework, new solutions are discovered through a neighborhood structure based on ejection chains. Numerical results show the benefits of these procedures in a real-time context. The impact of a masterslave parallelization scheme, using an increasing number of processors is also examined.

Chen et al.(2006) propose a realtimedependent vehicle routing problem with time windows. This problem is expressed as a series of mixed integer programming models that account for realtime and timedependent travel times, as well as for realtime demands in a unified framework. Vehicle routes and the departure times are treated as decision variables, with delayed departure allowed at each node .Also a route construction and a route improvement heuristics are proposed.

An efficient route minimization heuristic for the vehicle routing problem with time windows was proposed by Yuichi & Olli (2009). The heuristic proposed by them is based on the discharge pool, powerful insertion and guided local search strategies.Peng (2011)introduced a route construction heuristic with an adaptive parallel scheme is and the results from extensive computational experiments show the proposed parallel route construction heuristic is efficient and effective for routes construction, which is particularly useful for generation of the initial solutions for many meta-heuristic approaches with improved solution quality and convergence of the solution process. Patrıciaet al.(2009) proposed heuristics and a scatter search algorithms to solve reallife heterogeneous fleet vehicle routing problem with time windows and split deliveries. They proposed two constructive heuristics to generate the initial solutions of Scatter search alogorithm.

1.4.3MetaHeuristics Approaches

Many of the most successful meta-heuristics for the large VRPTW instances are based on some form of parallel computation. Blanton & Wainwright (1993) were applied a genetic algorithm to VRPTW. They crossed a genetic algorithm with a greedy heuristic. Under this scheme, the genetic algorithm searches for a good ordering of customers, while the construction of the feasible solution is handled by the greedy heuristic. During the past few years, numerous papers have been written on generating good solutions for VRPTW with Generalized Algorithms. According to the authors of Blanton & Wainwright (1993),Berger et al.(1998),local searches (Potvin &Bengio (1996),Thangiah et al.(1995)) and other meta heuristics such as tabu search Ho et al.(2001) and ant colony system (Berger et al.(2003) it represents hybridizations of a GA with different construction heuristics.

Hoonget al.(2003) have measured a variant of VRPTW constrained by a limited vehicle fleet, which is a more realistic problem in logistics. A limited number of vehicles is given (m-VRPTW). Under the particular scenario, a feasible solution is the one that may contain either unserved customers and/or relaxed time windows. It presents an analytical upper bound for that formulation, and show that their search approach came fairly close to the upper bound. With the stand point of the stability, this algorithm is more feasible. It also show that the same algorithm could be used to give reasonably good results for the standard VRPTW problem.

Montemanniet al.(2005) proposed an ant colony system (ACS) meta heuristic procedure to solve the DVRP. It is based on the partition of the working day into different time slots. A sequence of static vehicle routing problems is then generated. And used an Ant Colony System algorithm to solve the problems. The properties of ACS have been also exploited to transfer information about good solutions from a time slice to the following one. They define new public domain benchmark problems and they test the algorithms that they proposed on those bench mark instances. A computational study on a newly defined set of benchmarks, finally showed that the method proposed was able to achieve good results both on artificial and absolute problems.

Cho & Wang (2005) present a metaheuristic approach, which is based on the Threshold Accepting combined with modified Nearest Neighbour and Exchange procedures, to solve the Vehicle Routing Problem with Backhauls and Time Windows (VRPBTW). The VRPBTW accepts that trucks initially start from the depot, deliver goods to the linehaul customers, successively pickup goods from the backhaul consumers, and finally return to depot.Alba &Dorronsoro (2005) proposed a Cellular Genetic Algorithm (CGA) which is a kind of decentralized population based heuristic, which is used for solving CVRP, improving several of the best existing results so far in the literature. The study shows a high performance in terms of the quality of the solutions found and the number of function evaluations.

Bouthillier &Crainic (2005) they proposed a cooperative parallel metaheuristic for the VRPTW, based on the of a solution warehouse strategy. Distics search threads were identified :cooperate asynchronously, exchanging information on the best solutions . The exchanges are performed through a mechanism called solution warehouse, which holds and manages a pool of solutions. Homberger and Gehring (2005)developed a twophase hybrid meta heuristic for the VRPTW. The objective function of the VRPTW considered here combines the minimization of the number of vehicles (primary criterion) and the total travel distance (secondary criterion). The aim of the first phase is the minimization of the number of vehicles by means of a (l;k)-evolution strategy, whereas in the second phase the total distance is minimized using a tabu search algorithm.

Zhang & Tang (2009) present a novel hybrid ant colony optimization approach called SSACO (Scatter search ant cololny optimization) algorithm to solve the vehicle routing problem. The main feature of the hybrid algorithm is to hybridize the solution construction mechanism of the Ant Colony Optimization (ACO) with Scatter Search (SS). In the hybrid algorithm, ACO algorithm is usedheuristic to generate the initial solutions which are then formed the reference set. Within the scatter search framework, after twosolution combination method for the reference set has been applied, we employ ACO method to generate new solutions through updating the common arc pheromone mechanism. Moreover, during implementing the hybrid algorithm, cyclic transfers, a new class of neighborhood search algorithm can also be embedded into the scatter search framework as neighborhood search to improve solutions. Despite the size of the cyclic transfer neighborhood is very large, a restricted subset of the cyclic transfer neighborhood is adopted to reduce the computational requirements to reasonable levels.

A new and effective meta-heuristic algorithm, directed evolutional strategies, for the VRPTW that is presented by David &Braysy (2005). The algorithm combines the strengths of the guided local search and evolution strategies belonging to meta-heuristics into an iterative twostage procedure. Guided local search is used to regulate a compound local search in the first stage and the neighbourhood of the growth strategies algorithm in the second stage. Russell & Chiang (2006) have used a scatter search metaheuristics to solve the VRPTW. Both a common arc method and an optimization-based set covering model are used to combine vehicle routing solutions. A reactive tabu search metaheuristic and a tabu search with an advanced recovery feature, together with a set covering procedure are used for solution improvement.

Benoit et al.(2007) proposed a multidepot vehicle routing problem with interdepot routes, which addresses an extension of the multi-depot vehicle routing problem in which vehicles may be refilled at intermediate depots along their routes. Itproposed a heuristic approach combining the adaptive memory principle, a tabu search method for the solution of sub-problems, and integer programming.

A multi-objective Evolutionary Algorithm (EA) for solving the VRPTW was suggested by Abel &Bullinaria (2011). For multi-objective problems, heuristics generally have two objectives : (1) to minimize the distance of the generated solutions, called the Pareto approximation, from the true Pareto front, and (2) to maximize the diversity.

Evolutionary Algorithms (EAs) are optimizers based on Darwin's theory of evolution, where only the fittest individuals survive and produce offspring to populate the next generation.Aziz (2010) proposed a hybrid ACO algorithm for solving vehicle routing problem (VRP) heuristically in combination with an exact algorithm. In the basic VRP, geographically scattered customers of known demand are provided from a single depot by a fleet of identically capacitated vehicles. The intuition of the proposed algorithm is that nodes which are near to each other will probably belong to the same branch of the minimum spanning tree of the problem graph hence will probably belong to the same route in VRP. Given a clustering of client nodes, the solution is to find a route in these clusters by using ACO with a modified version of transition rule of the ants. At the end of each iteration, ACO tries to improve the quality of solutions by using a local search algorithm, and update the associated weights of the graph arcs.

Bin & Zhen (2011) have expressed an Improved Ant Colony Optimization (IACO) algorithm to solve the period vehicle routing problem with time windows (PVRPTW). In PVRPTW, the planning period is extended to several days and each customer must be served within a specified time window. Multi-dimension pheromone matrix is used to accumulate heuristic information on different days. Two cross-over operations are introduced to improve the performance of the algorithm.

Balseiro et .al(2011) suggested an Ant Colony algorithm hybridized with insertion heuristics for the Time Dependent Vehicle Routing Problem with Time Windows (TDVRPTW). In the TDVRPTW, a fleet of vehicles must deliver goods to a set of customers, time window constraints of the customers must be respected and the fact that the travel time between two points depend on the time of departure, has to be taken into account.

1.4.4 Hybrid Methods

Hybrid methods use a combination of exact, heuristic procedure or meta-heuristics to solve the VRP.Backer &Furnon (1997) have proposed a hybrid method using constraint programming and meta-heuristics. Constraint Programming naturally uses the technique of depth-first branch and bound as the method of solving optimization problems. Although this method can give the optimal solution for large problems, the time needed to find the optimal will be limited. This paper introduces a method for using iterative improvement techniques within a Constraint programming framework and applies this technique to vehicle routing problems. The introduced constraint programming model for vehicle routing, after which they describe a system for integrating constraint programming and iterative improvement techniques. Aftermath it described how the method can be greatly accelerated by handling core constraints using fast local checks, while other more complex constraints are left to the constraint propagation system. It has coupled their iterative improvement technique with a meta-heuristic to avoid the search being trapped in local minimal. It uses two meta-heuristics: a simple Tabu Search procedure and Guided Local Search. Ithave conducted an empirical study over benchmark problems and it shows the relative merits of these two techniques.

II. INTRODUCTION TO THE DISCRETE FACILITY LOCATION PROBLEM

Facility location models have been widely studied due to their application in many real situations. These models can differ in their objective function, the number of the services to locate, the solution space in which the problem is defined, and several other decision issues. The problem is called as a discrete facility location problem if there are a finite number of candidate facility locations. If the facilities can be placed anywhere in some continuous regions, then the problem is called as a continuous facility location problem. This work is focused on discrete facility location models. Discrete facility location problems (FLP) are concerned with choosing the best location for facilities from a given set of potential sites to minimize the total cost while satisfying customer demand. In the uncapacitated facility location problem (UFLP), each facility is assumed to have no limit on its capacity where each customer accepts all its demand from exactly one facility.

(Kuehn & Hamburger, 1963) and Balinski (1965) together proposed the first model called "Simple plant location problem - (SPLP)". (Hakimi (1964) and Hakimi (1965) defined P-Median Problem (PM). It was later shown that these two models were particular cases of a more general formulation for deterministic, static, uncapacitated problems having a minimum objective function (Cornuejols et al. (1977), Galvao& Raggi (1989)). The capacitated facility location problem (CFLP) is a well-known combinatorial optimization problem. It consists in determining which facilities to open from a given set of potential facility locations and how to assign customers to those facilities.

2.1 Uncapacitated facility location problem (UFLP)

Kuehn & Hamburger (1963) developed a heuristic computer program for locating warehouses by compairing it with recently published efforts at solving the problem either by means of simulation or as a variant of Linear Programming (LP). Erlenkotter (1978) developed one of the earlier bounding procedures, called dual-ascent and dual-adjustment, based on the LP relaxation of the problem. Guignard (1988) established a dual-ascent algorithm for solving Lagrangian dual problem obtained by relaxing constraints. Galvao and Raggi (1989) established a method for solving to optimality a general 0-1 formulation for UFLP. This is a three-stage method that solves large problems in reasonable computing times which is composed of a primal-dual algorithm, a sub gradient optimization to solve a Lagrangean dual and Branch Bound (BB) algorithm.

Goldengorin et al. (2004) presented a technique that enhances the performance of BB algorithms. The new algorithms thus obtained are called branch and peg algorithms, where pegging mentions for assigning values to variables outside the branching process. Sun (2006) developed a Tabu Search (TS) heuristic procedure to solve UFLP. Hansen et al. (2007) applied the Variable Neighbourhood Search Metaheuristic (VNSM) to the primal simple plant-location problem (SPLP) and to a reduced dual that is obtained by exploiting the complementary negligence conditions to solve some very large-scale Euclidean instances.

Kratica et al. (2014) considered the multi-level UFLP and new mixed integer linear programming (MILP) formulation is offered. Experimental results are performed on instances known from literature. Ardjmand et al. (2014) proposed a discrete variant of Unconscious search (US) which minimized the process of psychoanalytic psychotherapy in which the psychoanalyst tries to access the unconscious of a mental patient to find the root cause his/her problem, which is encapsulated in unconsciousness. Letchford& Miller (2014) presented an aggressive reduction scheme involving four different reduction rules, along with lower-and upper bounding procedures in BB. Monabbati&Kakhki (2015) presented a sub additive dual ascent procedure to find an optimal sub additive dual function based on Klabjan's generator sub additive function.

2.2 Capacitated facility location problem (CFLP)

Khumawala (1974) introduced an efficient heuristic procedure for solving a special class of mixed integer programming problem called the capacitated warehouse (plant) location problem. Geoffrion& McBride (1978), Nauss (1978) ,Christofides& Beasley (1983), Pirkul (1987) and Shetty (1990) considered Lagrangean Relaxation (LR) of the demand constraints with or without addition of an aggregate capacity constraint or another surrogate constraint for solving large scale CFLP. The algorithm consists of solving a SFLP and then solving a minimum cost network problem. The problem is solved by applying an iterative procedure in which lower bounds on the optimal objective value are generated through LR techniques. Upper bounds on the optimal objective value are obtained by modifying the Lagrangian solution to restore feasibility. However, Nauss (1978) also presented a Branch and Bound algorithm.

Magnanti and Wong (1981) proposed methodology for refining the performance of Benders decomposition when applied to mixed integer programs. Jacobsen (1983) generalized heuristics for CFLP that are ADD/DROP, SHIFT, ALA and VSM (Vertex Substitution Method). Domschke and Drexl (1985) noted that ADD-heuristics normally lead to bad solutions. They presented some starting procedures in order to overcome this difficulty. Van Roy (1986) presented an implementation of the Cross-Decomposition method to solve CFLP. The method unifies Benders Decomposition and LR into a single framework that involves successive solutions to a transportation problem and a SPLP.

Doong et al. (2007) used a hybrid method of genetic algorithm and sub gradient technique. However, (Klose &Görtz (2007)) employed a (stabilized) column generation method and then the column generation procedure within a branch-and-price algorithm for computing optimal solutions to the CFLP. Sambola et al. (2011) formulated the problem as a two-stage stochastic program and two different recourse actions were considered. Kucukdenizet al.(2012) proposed a fuzzy c-means clustering algorithm based method which involves the integrated use of fuzzy c-means and convex programming. Rahmani&MirHassani (2014) proposed a new hybrid optimization method called Hybrid Evolutionary Firefly-Genetic Algorithm which is inspired by social behavior of fireflies and the phenomenon of bioluminescent communication. The method combines the discrete Firefly Algorithm (FA) with the standard Genetic Algorithm (GA).

Harris et al. (2014) planned an efficient evolutionary multi objective optimization approach to the CFLP for solving large instances that considers flexibility at the allocation level, where financial costs and CO_2 emissions are considered simultaneously. Ozgen&Gulsun (2014) combined a two-phase possibilistic LP approach and a fuzzy Analytical Hierarchical Process (AHP) approach to optimize two objective functions (‘‘Minimum cost’’ and ‘‘Maximum qualitative factors benefit’’) in a four-stage (suppliers, plants, distribution centers, customers) supply chain network in the presence of vagueness. Li et al. (2014) studied multi-product facility location problem in a two-stage supply chain in which plants have production limitation, potential depots have limited storage capacity and customer demands must be satisfied by plants via depots. They developed a hybrid method. Aardalet al.(2015) gave the first Fully polynomial time approximation scheme (FPTAS) for the single-sink (single-client) CFLP. Then, they showed that the problem is solvable in polynomial time if the number of clients is fixed by reducing it to a collection of transportation problems.

2.3 Single source capacitated (Multi) Facility Location Problem (SSCMFLP)

As a special case of CFLP, discrete facility location problems can be considered. There is a vast amount of research interest in literature devoted to the Single Source Capacitated (Multi) Facility Location Problem (SSCMFLP).

Nagelhaut& Thompson (1980) gave two heuristic solution methods and BB algorithm for solving single source transportation problems. Neebe& Rao (1983) reformulate the SSCFLP as a set partitioning problem. They proposed a column generation and branch-and-price method to solve it.

Klincewicz&Luss (1986) described a LR heuristic algorithm. By relaxing the capacity constraints, the uncapacitated facility location problem is obtained as a subproblem and solved by the well-known dual ascent algorithm. Darby-Dowman et al. (1988) have considered SSCFLP by proposing the use of LR in which the capacity constraints were relaxed. Sridharan (1993) proposed a heuristic based on a LR. Tranganlaterngsak et al. (1997) proposed a facility location problem in which there exist two echelons of facilities. Each facility in the second echelon has limited capacity and can be supplied by only one facility in the first echelon. Ronnqvist et al. (1999) proposed an approach which is based on a repeated matching algorithm which essentially solves a series of matching problems until certain convergence criteria are satisfied.

Hindi &Pienkosz (1999) combined LR with restricted neighbourhood search. However, Tragantalerngsaket al.(2000) proposed a LR based BB algorithm for its solution. Cortinhalet al.(2003) used a LR to obtain lower bounds for this problem. Upper bounds are given by Lagrangean heuristics followed by search methods and by one TS metaheuristic. Guastaroba& Speranza (2014) extended the Kernel Search heuristic framework to general Binary Integer Linear Programming (BILP) problems and apply it to the SSCFLP. Dantrakul et al. (2014) introduced three methods (greedy algorithm, PM algorithm and p-center algorithm) to solve the problem. Bieniek (2015) studied SSCFLP with general stochastic identically distributed. They unified a priori solution for the locations of facilities and for the allocation of customers to the operating facilities is found and a deterministic equivalent formulation of the model is presented.

2.3.1 General Mathematical Formulation for Single source capacitated (Multi) Facility Location Problem (SSCMFLP)

Let I={1,…,n} be a set of potential locations and J={1,…,m} be a set of customers. Each customer j has an associated demand, w_i, that must be served by a single facility. Facilities can be located only at some prespecified sites in a plane and there is a limit, s_i, to the total demand that a facility located at a site i can meet. The costs c_{ij} of supplying the demand of a customer j from a facility established at location i, as well as the fixed costs f_i for opening a facility at i, are known. It should be noted that the distances values between customers and facilities situated at potential sites are embedded in supplying costs c_{ij} Let

$x_i =$ $x_{ij} = 1$, if customer j is assigned to a facility located at i
$x_{ij} = 0$, Otherwise

$y_i = 1$, if a facility is located at candidate site i
$y_i = 0$, Otherwise

Then, SSCMFLP can be mathematically stated as Holmberg et al. (1999) ;

$$Min \sum_{i \in I} \sum_{j \in J} \left(c_{ij}\, x_{ij} \right) + \sum_{i \in I} \left(f_i y_j \right) = 0 \;------------ (1)$$

Subject to

$$\sum_{j \in J} \left(w_j x_{ij} \;\le\; s_i y_i \right) = 0 \qquad \forall\, i \in I \quad ------------- (2)$$

$$\sum_{i \in I} \left(x_{ij} = 1 \right) \qquad \forall\, j \in J \quad ----------------- (3)$$

$$x_{ij} \in \{0\,,1\}, \;\; \forall\, i \in I \quad ,\forall\, j \in J \;--------------- (4)$$
$$y_i \in \{0\,,1\}, \;\; \forall\, i \in I \;\; -=- ------------------ (5)$$

For a single-source assumption FLPs, the objective functions (1) calculate the total transportation. Constraints (2) ensure that the capacity of each facility is not exceeded while constraints (3) guarantee that every customer must be served from exactly one facility.

2.4 Facility Location Models

In this section provide the most used mathematical models for FLPs. The models are defined in a discrete location space, namely have a set of demand points (customers) with an associated weight and a set of facility location that are the possible position for the new facility that want to locate. Between customer iand potential facility j, evaluate a distance, in some way among them indicated previously.

2.4.1 Median Problems

The p-median problem aims the minimization of the weighted sum of the distances between p facilities to be opened and a set of demand points. Several versions of the problem have been defined in the literature, and it has been used in many different applications varying from the location of industrial plants and warehouses or public facilities by Revelle&Eiselt (2005) but also as a tool for data mining applications Ng & Han (1994). In the model there is the important assumption that we have to locate exactly p facilities, it is a situation considerable very near to the practice as when we know, for example, a budget constraint for the number of facilities to locate. The other important consideration is that the customers will be located to the nearest facility.

So we introduce the follow notations, common for the three proposed models:
I = set of demand points,
J = set of possible locations for the facilities,
d_{ij}= distance between customer i and potential facility j,

p = total number of facilities to located,
h_i =weight associated to each demand point (demand or number of customers).

We define the, allocation decisions, namely which facility j satisfy the demand expressed by a customer i, through the following x-variables:

x_{ij}= 1 ,if demand point iis allocated to facility j
x_{ij}= 0 , otherwise $\forall \quad i, j \in J$
and location decisions, are represented with
y_{ij}= 1 ,if a facility is located at point j
y_{ij}= 0 , otherwise $\forall \quad j \in J$

The formulation proposed by Hakimi (1964) and Revelle&Swaim (1970) retained the model as following way.

Min $\sum_{i \in I, j \in J,} h_i \, d_{ij} \, x_{ij}$

$$Such\ that$$

$$\sum_{j \in J,} x_{ij} = 1 \quad \forall\, i \in I, \quad --------------------- (1)$$

$$(x_{ij} \leq y_j) \ , \forall\, i \in I \ , \forall\, j \in J, \ i \neq j \ ------------- (2)$$

$$\sum_{j \in J,} y_j = p \quad --- ---------------------- (3)$$

$$x_{ij} \ , \ y_j \in \{0,1\}, \ \forall\, i \in I \ , \forall\, j \in J \ ----------------- (4)$$

Constraints (1) ensure that all the demand points are allocated. Constraints (2) guarantee that a point receives allocation only if it is a plant. Constraint (3) fixes the number of plants to p. Constraints (4) states that all variables are binary. It is classified as NP-hard (Kariv& Hakimi (1969) so for solving it we find in literature a very huge number of exact method and metaheuristic approaches that look for a good solution (sometimes the optimal solution) when the problem is characterized by a big number of demand points and facilities. A very comprehensive survey about heuristic approaches for median problems is provided by Mladenovic et al. (2007) that resumes both classical heuristic methods than metaheuristic approaches, giving also important indication of the instances used in the literature for testing them. For what concerning the exact method we can indicated, among others, the landmark study of Beasley (1969), Galvao& Raggi (1969). There also a lot of extension to the problem like the capacitated version studied for example by Mulvey & Beck (1984) and recently by Lorena &Senne (2004), or the generalized version with more than one type of facility called Multi-Weber problem (Cooper (1963) , Cooper (1964)). The MiniSum objectives is also used in the other well-known model called Simple Plant Location Problem (Erlenkotter, 1978) where the number of facilities to fix is a variable of the problem.

2.4.2 Center Problems

An important class of problem owned to MiniMax problem are the p-center problems. The p-center problem seeks to minimize the maximum distance between any demand and its nearest facility. At difference of the previous class we want that the maximum distance between a demand point and its closest facility is as small as possible instead to minimize the total distance between demand points and facility. Indicating with " D " the maximum distance between a demand node and the nearest facility, and using the same variables of the previous case the problem can be formalized as follows Hakimi (1964)

Min D

Such that,

$$\sum_{j \in J,} x_{ij} = 1 \quad \forall\, i \in I, \quad ------- ------------- (1)$$

$$(x_{ij} \leq y_j) \ , \forall\, i \in I \ , \forall\, j \in J, \ i \neq j \ ---------------- (2)$$

$$\sum_{j \in J,} y_j = p \quad --- --------------------(3)$$

$$D \geq \sum_{j \in J,} (d_{ij})(x_{ij}) \quad \forall\, i \in I, \quad --------- (4)$$

$$x_{ij} \ , \ y_j \in \{0,1\}, \ \forall\, i \in I \ , \forall\, j \in J \ --------------(5)$$

The objective function is to minimize the maximum distance between any demand node and its nearest facility. Constraints (1) to (3) are identical to the p-median problem. Constraint (4) defines the maximum distance between any demand node iand the nearest facility j. Finally, constraints (5) are binary constraints for the decision variables. If the number of services to located is equal to 1 we call the problem Absolute center Problems Hakimi (1964). In some cases, at each demand point is also associated a weight Daskin (1995) and the objective function become:

$$D \geq h_i \sum_{j \in J,} (d_{ij})(x_{ij}) \quad ------------------ \quad \forall\, i \in I,$$

If facility locations are restricted to the nodes of the network, the problem is a vertex center problem Daskin (1995). For example, Burkard &Dollani (2007) formalized a 1-center problem on a network with positive and negative vertex weights with the objective to minimize a linear combination of the maximum weighted distances of the center to the vertices with positive weights and to the vertices with negative weights. Ozsoy& Pinar (2006) introduced the capacity restrictions on the facilities. Moreover, a lot of applications can be described with center models. Biazaran&SeyediNezhad (2009) summarized the possibilities in the location of emergency services, like hospitals and fire stations and computer network services like location of the data files; but also, in the distribution system, or for military purpose and public facilities like parks, post boxes and bus stops.

2.4.3. Covering Problems

One of the classical objectives in location modeling is “coverage” which seeks to ensure that each customer is "covered" namely attend by a certain facility if the distance between them is lower than a certain threshold or required distance. The first model of this type was proposed by Church &ReVelle (1974) and it is called the p-Maximal Covering Location Problem. It consists in locating p facilities that can cover the maximum amount of demand.

The proposed formulation and model as follows.

z_i= 1 ,if customer iis covered by some facility
z_i= 0 , otherwise $\forall \quad i \in I$
A possible formulation for the thep-Maximal Covering Location Problem is:

$$\text{Max} \ \sum_{i \in I,} (h_i)(z_i) \quad ---------------- \quad (1)$$

$$\textit{Such that}$$

$$\sum_{j \in J,} y_j \geq z_i \quad \forall\, i \in I, ---------------- \quad (2)$$

$$\sum_{j \in J,} y_j = p \quad ------------------------ \quad (3)$$

$$z_i,\ y_j \in \{0,1\}, \ \forall\, i \in I \quad , \forall\, j \in J \ ---------------- \quad (4)$$

Constraints (2) guarantee that a point can covered only by facility opened. Constraints (3) fixes the number of plants to p. Constraints (4) states that all variables are binary.

The most important Set Covering Location Problem introduced by Hakimi (1964) and formulated as integer programming by Toregas et al. (1971) that consists in founding the minimum number or the minimum cost set of facilities such that every demand point is covered by some facilities. In addition, there are several covering problems born from modulating in different way the covering concept. Then can have the back-up coverage, in which demand points are required to be covered by more than one open facility. Storbeck (1982) in his model maximized, in addition to the demand covered by the facilities, also the demand covered by at least two facilities. Moreover Daskin et al. (1988) formulated a problem in which has to be maximized only the back-up coverage.

Church and Roberts (1984) and Karasakal (2004) successively introduced the concept of gradual or partial coverage where for each facility has two covering radius: a minimum covering radius and a maximum covering radius; demand points within the minimum radius are considered to be totally covered, while the ones falling in the area between the circles described by the two radii are considered to be partially covered.

III. INTRODUCTION TO LOCATIONAL ROUTING PROBLEMS

It is normally recognized that logistic costs consume a large part of the budget of companies. These costs can be significantly reduced by a careful design of the supply chain. The distribution network at the end of the chain is particularly important because it involves many small product flows towards end-customers or retailers. The design of this network raises two hard combinatorial optimization problems, to locate depots and determine vehicle routes supplying customers from these depots. These two types of decisions have been addressed separately for a long time, but the continuous progress in optimization techniques has made possible integrated approaches known under the name of locationrouting problems (LRP).

The idea of combining depot location and vehicle routing dates back nearly fifty years. At that time, the inter-dependency of these two types of decisions was already highlighted but optimization and computers were not developed enough to envisage an integrated treatment (Maranzana, 1964; Von Boventer, 1961; Webb, 1968). Watson-Gandy and Dohrn (1973) were probably the first authors who clearly considered customer visits while locating depots, through a non-linear profit function modeling decreasing sales with the distance to the depot.

This key-remark inspired a growing stream of research on location-routing problems and the last comprehensive survey published by Nagy and Salhi (2007) demonstrate its vitality. Nagy and Salhi classify the literature on location-routing problems by distinguishing methodologies for deterministic variants, stochastic or dynamic problems, and versions with more complex networks, before concluding with some suggestions for future research.

Melkote and Daskin (2001) presented accurate mathematical models and effective solution techniques to face location, allocation and distribution problems (Caramia et.al (2007),Nagy&Salhi (2005)) that resort to the concept of integrated logistics systems and whose basis is constituted by a combined location-routing model(Min (1996),Nagy andSalhi (2007)). The main difference between a location-routing problem (LRP) and a classical location-allocation problem is that, once the facility is located, in the former it's required that the customers are served along a route, while in the latter, every customer is directly connected to the same facility (Klose A and Drexl (2005) , Min et al.(1998)).

However, the LRP, considered as "the scheduling of locations taking in account tour scheduling issues", is, clearly, NP-hard since it is constituted by two NP-hard problems.

On the other hand, location and routing problems can be seen as special cases of LRP:

(i) if each customer is directly connected to the facility, the LRP reduces to a classical location problem;

(ii) if the Centre location is settled, the LRP can be considered as a VRP.

Solution methodologies can be classified according to the way that, they create a relationship between the location and routing problems. (Ropke and Pisinger (2006), Wade and Salhi (2002)).

In Sequential methods the location problem is first solved by minimizing the distances between facility and consumers, then a routing problem is faced. These methods don't allow a feedback from the routing phase to the location one. So, a sub-optimal design for the distribution system could be determined. Clustering solution methods first divide and group the customers, then:

(I) for each cluster a facility is located and a VRP (or TSP) is executed (Toth & Vigo) (1999)

(II) a TSP for each cluster is executed and then the facilities are located.

Iterative heuristics decompose the problem in twosub-problems which are iteratively solved moving the data from a phase to the other. Although iterative methods are an improvement of sequential methods, when the location algorithm ends, it starts again receiving as input the new information coming from the routing algorithm. From a designing point of view, iterative heuristics give the same importance to thesesub-problems. Hierarchical heuristicsconsider, instead, the location as the principal problem and the routing as a subordinate problem. To solve an LRP it is possible to use multi-phase-based procedures, which, break-up the problem, reduce its complexity

3.1 Problem definition

Guerra et.al (2007) developed innovative approach for the Location - Routing Problem (LRP). A set of consumers and potential facility is given. If d_i is the demand of a consumer, each consumer with $d_i>0$ must be allocated to a facility to completely satisfy d_i. The consignment is delivered through vehicles that depart from a facility and operate on circuits that include more customers. The set-up cost of a centre and the unitary distribution cost have been fixed. The vehicles and the potential centres have limited capacity. Facilities location and vehicles routes have to be determined, so to minimize the overall costs. (location and distribution costs).

The CLRP is bound by the followings conditions:

- The demand of each customer must be satisfied;
- Each customer must be served by a single vehicle;
- The overall demand on every route must be smaller or, at the most, equal to the capacity of the vehicle allocated to the route;
- Each route begins and ends to the same facility.

It is assumed, moreover, that the vehicle fleet is homogeneous and there's no limit to its dimension.

3.2 Proposed model of the problem

Let G = (V,A) be an oriented graph, where V = {v_1, …….v_{m+n}}is constitute by the nodes F = { v_1, …….v_m) (potential facility location) and by the nodes I ={v_{m+1},......V_{m+n}} (demand centres). Each edgeV_i V_j∈A represents the existing link between the pair of nodes that defines it and it is associated with a distance, or cost, C_{ij}>0. If some connections between nodes are forbidden, it is still possible to consider a complete graph setting to ∞the distance between them. It's assumed the graph to be symmetrical, therefore $C_{ij} = C_{ji}$. For each potential service node v_j∈I it is known the maximum service capacity Q; for each demand node vj∈I it is known the service demand d_i. The deliveries are affected by a fleet of k vehicles characterized by a maximum capacity K. If D is the set of potential facilities, I is thecustomers set, V the vehicles set, the mathematical formulation of the problem is the following:

$$Min \sum_{i\in D} (F_i y_i) \sum_{k\in V} \sum_{i\in D\cup U} \sum_{j\in D\cup U, j\neq i} (C_{ij} X_{ijk}) ----- \quad (1)$$

Subject to

$$\sum_{k\in V} \sum_{i\in D\cup I} (x_{ijk}) = 1 \qquad \forall\, j \in I \quad ------------(2)$$

$$\sum_{i\in D} \sum_{j\in I} (x_{ijk}) \leq 1 \qquad \forall\, k \in V \quad ------------(3)$$

$$\sum_{j\in D\cup I} (x_{ijk}) = \sum_{j\in D\cup I} (x_{jik}) \forall\, k \in V, \forall\, i \in I \cup D \quad --------(4)$$

$$\sum_{j\in I} \sum_{i\in D\cup I} (d_j x_{ijk}) \leq Q_k \qquad \forall\, k \in V \quad ---------- \ (5)$$

$$\sum_{j\in I} d_j z_{ij} \leq V_i y_i \qquad \forall\, i \in D \quad ------------- \ (6)$$

$$\sum_{j\in I} \sum_{k\in V} (x_{ijk}) - y_i \geq 0 \qquad \forall\, i \in D \quad ---------- \ (7)$$

$$\sum_{j\in I} x_{ijk} - y_i \leq 0 \qquad \forall\, i \in D, \forall\, k \in V \quad ----------(8)$$

$$\sum_{i\in I} (x_{ijk}) = \sum_{i\in I} (x_{\mathrm{jki}}) \forall\, \mathrm{j} \in \mathrm{D}, \forall\, \mathrm{k} \in \mathrm{V} - \ ---------- \ (9)$$

$$\sum_{k\in D}\sum_{i\in S}\sum_{j\in S}(X_{ijk}) \leq |S| - \left[\sum_{p\in S}\frac{|d|}{Q} = \right] \forall S \in I, |S| \geq 2 - (10)$$

$$x_{ijk} \in \{0,1\}\ \forall\, i,j \in I \cup D - - - - - - - - - - - - - - - - - - (11)$$
$$y_i \in \{0,1\},\quad \forall\, i \in D - (12)$$
$$z_{ij} \in \{0,1\},\quad \forall\, j \in I, \forall\, i \in D - - - - - - - - - - - - - - - - - - (13)$$

Where

- ✓ F_i = set up cost for facility i ,i ∈ D
- ✓ C_{ij} = edge i-j cost, i , j ∈ D ∪ I
- ✓ d_j = demand of the customer j, j ∈ I
- ✓ V_i = capacity of the facility i ,i ∈ D
- ✓ Q_k = capacity of vehicle k ,k ∈ V
- ✓ x_{ijk} = 1 , if vehicle k goes from i node to j node , i , j ∈ D ∪ I , k ∈ V
- ✓ S = {D} ∪ {I} set of all possible facility locations and customers
- ✓ y_i = 1 , if a facility is set up at node i , i ∈ D
- ✓ z_{ij} = 1 , if customer j is allocated to facility i, j ∈ I , i ∈ D

The objective function minimizes the set-up costs of the facilities and the distribution costs. Constraint (2) guarantees that each customer has been assigned to a single facility, Constraint (3) guarantees that each vehicle is sent by a single depository. Constraint (4) assures that the very same vehicle enters and exits in each node i, Constraint (5) and (6) assure that vehicle and facilities capacities are not exceeded. Constraint (7) and (8) assure that vehicles only come from opened facilities and Constraint (9) assures that a vehicle leaves and arrives in the same facility. Constraint (10) guarantees the absence of sub-circuits.

IV. CONCLUSION

The distribution cost is one of the main components of logistics costs, the effect is at a larger extent towards the total cost structure of an organization. To make the process optimized, an arrangement of vehicles and routes is one of the key activities.Thus, Vehicle Routing Problem forms an integral part of supply chain management, for productivity improvement in organizations through efficient and effective delivery of goods / services to customers. The paper aims in surveying the recent developments of Vehicle Routing Problem (VRP) and its variants. The review of literature has done under several areas: exact methods, heuristics approaches, meta-heuristics, and hybrid methods. Further, the contribution made by different researchers to the filed have been discussed in detail.The paper consists reviewing fundamental models in location theory and the complex applications in many sectors: distribution planning systems, telecommunication network designing, supply chain decisions etc. While most of the literature has focused on inventory controlling within the warehouse considering mini applications of travel plans without having an overall plan, young researchers are planned to encourage in formulating solutions to address the issue of transportation from the warehouse to mini-hubs and from them to the retailers.

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