MAT 353/APS 421 2.0 Computational Mathematics
Worksheet/Semester 1-2017

Note: *The following problems may help you to develop your programming abilities in both MATLAB and Maple software.*

1. Write a MATLAB script to calculate the following summation using nested for loops:

   $$[3\cdot(2+1)]+[4\cdot(3+2+1)]+[5\cdot(4+3+2+1)]+\ldots+[1000\cdot(999+\ldots+1)].$$

2. Write a MATLAB script to generate a matrix that has elements shown below (without typing the numbers explicitly):

   $$A=\begin{pmatrix}12 & 8 & 4 & 0 & -4\\ 14 & 10 & 6 & 2 & -2\\ 16 & 12 & 8 & 4 & 0\end{pmatrix}.$$

3. Given a mathematical function $f(x)=x^2-sin(x)$, write a MATLAB function to calculate approximate area underneath the curve from $x=x_0$ to $x=x_1$. Test your function when $x_0=5$ and $x_1=1000$.

4. Write a MATLAB function to compute and plot the electrostatic force F between two charged particles, as a function of the distance r between them.
   According to Coulombs law,

   $$F=k_c\frac{q_1q_2}{r^2}\text{ Newtons,}$$
   $$k_c=\frac{1}{4\pi\varepsilon_0},$$
   $$\varepsilon_0=8.854\times10^{-12},$$

   where q_1 and q_2 are the charges on the two particles. If either particle is an electron, then $q_1=q_2=1.602\times10^{-19}$ Coulombs. The input to the function is a vector of distances r, and the output is to be a vector of the same size, containing the corresponding forces F. Test your function for $r=[3,5,6,8,10]$.

5. Write a Maple procedure **PrimeGap(n)** which returns the first two consecutive primes which are at least n apart. So **PrimeGap(100)** should return 370261,370373, since these are the first two consecutive primes which are at least 100 apart.

6. The Fibonacci polynomials $F_n(x)$ are defined by

$$F_0(x) = 1,\ F_1(x) = x,\ F_n(x) = xF_{n-1}(x) + F_{n-2}(x),\ \text{for } n \geq 2.$$

   Write a Maple procedure **FibonacciPolynomial(n)** which returns $F_n(x)$. Make sure it gives $F_6(x) = x^6 + 5x^4 + 6x^2 + 1$, not

$$x(x(x(x(x^2+1)+x)+x^2+1)+x(x^2+1)+x)+x(x(x^2+1)+x)+x^2+1.$$

   What is the coefficient of x^{11} in $F_{25}(x)$?

7. Write a Maple program using procedures to calculate your expecting overall Grade Point Average (GPA). Your program should consists the following sub-procedures
   (a) to convert grades to GPA,
   (b) to calculate overall GPA at end of each semester,
   (c) to display your results.

8. Write a MATLAB program to compute the value of π using following series for finite terms N

$$\frac{\pi^2 - 8}{16} = \sum_{n=1}^{N} \frac{1}{(2n-1)^2(2n+1)^2}$$

   You have to use at least the following functions in your program to
   (a) calculate the partial sums,
   (b) display the convergence,
   (c) determine the number of terms to get the accuracy of 10^{-5} (take the true value as π by chopping after 6 digits),
   (d) display the results.

9. Write a MATLAB script file to illustrate the nature of the roots for a given quadratic formula and display its roots. (*Hint* : use inbuilt function **solve()**)

10. Write a Maple program using procedures to take an array with positive integers as an input and display the following.

    (a) Summation of odd and even integers separately

    (b) Arithmetic mean of odd and even integers separately

11. Write a Maple program using procedures to calculate the surface area and the volume of a sphere. Write three different sub procedures to do the following

    (a) calculate area

    (b) calculate volume

    (c) display the results

12. Write a MATLAB function file for base conversion of a given positive integer. Write separate functions for following.

    (a) Convert a given decimal number into binary

    (b) Convert a given decimal number into any number base

13. The exponential of a real number x can be obtained using the following infinite sum:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^n}{n!} + ...$$

    (a) Write an iterative code (using loops) in MATLAB function file to calculate e^x for given x and finite terms n

    (b) Write a MATLAB recursive function file for the same problem

14. Write a Maple program to propose **a more suitable system of election** for Parliamentary elections in Sri Lanka (A proper system to select members to the Sri Lankan Parliament). You must propose your system based on the following observations.

(a) A mixed system, a hybrid of the *first past the post* system and the *proportional* system.

(b) The new system should ensure the establishment of a *stable government* and a *strong opposition*, *equitable representation* to numerically minority parties and communities, *closer nexus* between voters and their elected representatives and the democratic representation of the peoples mandate.

(c) The proposed system of elections should give weight to the *elimination or minimizing* violence, *undue* expenditure at elections and misappropriation of state resources at the time of elections.

(d) The system proposed should further be *easy* to comprehend and relatively *easy* to administer.

(e) Preferential voting system should *completely* be eliminated from the system.

15. The Fibonacci numbers F_i are defined by $F_1 = 1, F_2 = 1,\ \ F_i = F_{i-1} + F_{i-2}$, for $i \geq 3$, and the Fibonacci polynomials $F_i(x)$ are defined by $F_1(x) = 1, F_2(x) = 1,\ \ F_i(x) = xF_{i-1}(x) + F_{i-2}(x)$, for $i \geq 3$. Write a **Maple procedure** that consists of sub-procedures to obtain,

    (a) a sequence of Fibonacci polynomials $F_i(x)$, where $i \in \mathbb{Z}^+$.

    (b) a sequence of Fibonacci Numbers F_i, where $i \in \mathbb{Z}^+$. (You may use the result of Part (a)).

    (c) a sequence of the ratios of successive Fibonacci numbers found in Part (b).

    (d) a sequence of numbers a_i, $i \in \mathbb{Z}^+$, where a_i's (Continued Fractions) are given by

    $$a_1 = 1,\ \ a_i = 1 + \frac{1}{a_{i-1}} \text{ for } i \geq 2.$$

    Comments on the results of Part (c) and Part (d).

16. Write a **MATLAB function** called **PascalTriangle** to produce Pascal's Triangle for arbitrary number of rows and to identify some of

its number patterns. Your program should consist of following sub-functions.

(a) A function **myFactorial** that returns the factorial when a non-negative integer is received as input (*Hint: Consider using Matlab's **prod** function*). The factorial of non-negative integer n is defined as follows.

$$n! = \begin{cases} 1, & \text{if } n = 0, \\ n(n-1)(n-2)\cdots 1, & \text{if } n \neq 0. \end{cases}$$

(b) A function **myBinomial** that takes two inputs, n and k, then returns the binomial coefficient (*you may use the myFactorial function written in Part (a)*). The binomial coefficient is defined by

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

(c) A function **myPascalTriangle** to display Pascal's Triangle as a lower triangle of a square matrix for an arbitrary number of rows (*you may use the myBinomial function written in Part (b)*). The nth row of the Pascal's Triangle holds the binomial coefficients

$$\binom{n}{k}, k = 0, 1, \ldots, n.$$

(d) A function **RowSumsPascal** to find the row sums of the Pascal's Triangle (*you may use the myPascalTriangle function written in Part (c)*). Identify the pattern of the sums?

(e) A function **Pascal5by5** to display the five by five matrix form Pascal's Triangle such that having first five rows and first five columns (*you may use the myPascalTriangle function written in Part (c)*). Let p_i, $i = 1, ..., 5$ be any number given by putting a row elements together (for an example the third row [1,2,1] of Pascal's Triangle can be written as 121), verify that $p_i = 11^{i-1}$ for $i = 1, 2, ..., 5$.

17. Let n be a non-negative integer.

(a) Write a Maple procedure *BinomialExpansion(n)* which returns the expansions of $(x+y)^k$ for $k = 0 \ldots n$, where x and y are real variables.

(b) Write a Maple procedure *PascalsTriangle(n)* which returns the numerical *coefficients* of the expansions of $(x+y)^k$ for $k = 0 \ldots n$.

Test your procedures for few n values.

18. The *Hailstone* iteration is defined as follows: *given a positive integer n, repeatedly replace it either with* $n/2$ *(if it's even), or with* $3n+1$ *(if it's odd). Stop when you get to* 1. For example, if we take $n = 11$ we get the Hailstone sequence as,

$$11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.$$

The *Hailstone length* of n is the number of terms in its Hailstone sequence. For example, the Hailstone length of 11 is 15. Write a Maple procedure *HailstoneLength(n)* which returns the Hailstone length of n. Which value of n $(8 \leq n \leq 12)$ has the largest hailstone length?

19. The surface area A and the volume V of a cylinder of radius r and height h are given by the formulas $A = 2\pi r(h+r)$ and $V = \pi r^2 h$ respectively. Write a MATLAB function named *CylindersAreasVolumes(vr, vh)* that takes a vector of radii (vr) and a corresponding vector of heights (vh) of some finite number of cylinders as parameters and returns the surface areas and volumes of the cylinders using the formulas given above. Your program should consists *three different* sub functions to:

(a) calculate the area of a cylinder,

(b) calculate the volume of the cylinder, and

(c) display the results.

Test your program for $vr = [2, 5, 3, 6, 4]$ and $vh = [3, 4, 5, 7, 4]$.

20. The number e is an important mathematical constant that is the base of the natural logarithm. Write a MATLAB function *evalue(n)* to calculate approximate values of e for a given positive integer n using

(a) the series sum

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{n!} + ..., \text{ and}$$

(b) the limit, $\lim_{x \to n} (1 + 1/x)^x$, where x is a real variable.

Assuming true value of e is given by 2.7183, calculate the errors involve in both approximations for few n values.

21. (a) Write a Maple procedure called **mean** that computes the mean value of a list of numbers.

    (b) Write a Maple procedure called **variance** that computes the variance of a list (L) of numbers, i.e. **variance**(L) computes as

    $$\frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2,$$

    where n is the number of elements of the list L and μ is the mean value of the numbers in list L.

    (c) The z-scores of a set of numbers can be calculated from the formula $z = \frac{x - \mu}{\sigma}$, where z is the z-score, x is the value of the element, μ is the population mean, and σ is the standard deviation. Write a Maple procedure called z**score** that computes the z-scores of a list of numbers. Test your procedures in Part (a), Part (b) and Part (c) for the list of numbers $[1,2,3,4,5]$.

22. Let $L[n](x)$ denotes a special kind of the Laguerre polynomial of degree n in the variable x. We define $L[n](x)$ by $L[0](x) = 1, L[1](x) = x$, and
    for any degree $n > 1$;

    $$L[n](x) = \frac{(2 * n - 1 - x) * L[n-1](x) - (n-1) * L[n-2](x)}{n}.$$

    Write a Maple procedure called **Laguerre** that returns a sequence of Laguerre polynomials $L[i](x)$, where $i \in \mathbb{Z}^+$. Test your procedure for $i = 10$. What is the coefficient of x^2 in $L[10](x)$?

23. Write a MATLAB function called **Cosinevalue** that gives the approximate value of the cosine of a number x using the following relation;

$$cos(x) \approx \sum_{k=1}^{n+1} (-1)^{(k-1)} \frac{x^{2(k-1)}}{(2(k-1))!}.$$

Your function should consist of the sub function called **myfactorial** that returns the factorial when a non-negative integer is received as an input (**Hint**: You may use MATLAB built-in function *prod*). When $n = 9$, what is the error for $x = 2\pi$?

24. (a) The midpoint rule to compute a definite integral evaluates the function at the midpoint of the integration interval:

$$\int_a^b f(x)dx \approx (b-a)f\left(\frac{a+b}{2}\right).$$

Write a MATLAB function called **midpoint**(f,a,b) that returns the integral of f over the interval $[a,b]$ using the midpoint rule.

(b) Simpson's rule approximates a definite integral within the interval $[a,b]$ of function $f(x)$ as follows:

$$\int_a^b f(x)dx \approx \frac{b-a}{6}\left(f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right).$$

Write a MATLAB function called **Simpson**(f,a,b) that returns the integral of f over the interval $[a,b]$ using the Simpson's rule.

Test your both programs in Part (a) and Part (b) for the function $f(x) = -x^2+2x+5$ in the interval $[-2,2]$.