CS 3182 Modeling and Simulation
Worksheet-2018

The following problems may help you to develop your programming abilities in MATLAB software.

1. Write a MATLAB script to calculate the following summation using nested for loops:

   $$[3\cdot(2+1)]+[4\cdot(3+2+1)]+[5\cdot(4+3+2+1)]+\ldots+[1000\cdot(999+\ldots+1)].$$

2. Write a MATLAB script to generate a matrix that has elements shown below (without typing the numbers explicitly):

   $$A=\begin{pmatrix}12 & 8 & 4 & 0 & -4\\ 14 & 10 & 6 & 2 & -2\\ 16 & 12 & 8 & 4 & 0\end{pmatrix}.$$

3. Given a mathematical function $f(x)=x^2-sin(x)$, write a MATLAB function to calculate approximate area underneath the curve from $x=x_0$ to $x=x_1$. Test your function when $x_0=5$ and $x_1=1000$.

4. Write a MATLAB function to compute and plot the electrostatic force F between two charged particles, as a function of the distance r between them.
   According to Coulombs law,

   $$F=k_c\frac{q_1q_2}{r^2}\text{ Newtons,}$$
   $$k_c=\frac{1}{4\pi\varepsilon_0},$$
   $$\varepsilon_0=8.854\times10^{-12},$$

   where q_1 and q_2 are the charges on the two particles. If either particle is an electron, then $q_1=q_2=1.602\times10^{-19}$ Coulombs. The input to the function is a vector of distances r, and the output is to be a vector of the same size, containing the corresponding forces F. Test your function for $r=[3,5,6,8,10]$.

5. Write a MATLAB program to compute the value of π using following series for finite terms N

$$\frac{\pi^2-8}{16} = \sum_{n=1}^{N} \frac{1}{(2n-1)^2(2n+1)^2}$$

You have to use at least the following functions in your program to

(a) calculate the partial sums,

(b) display the convergence,

(c) determine the number of terms to get the accuracy of 10^{-5} (take the true value as π by chopping after 6 digits),

(d) display the results.

6. Write a MATLAB script file to illustrate the nature of the roots for a given quadratic formula and display its roots. (*Hint*: use inbuilt function **solve()**)

7. Write a MATLAB function file for base conversion of a given positive integer. Write separate functions for following.

(a) Convert a given decimal number into binary

(b) Convert a given decimal number into any number base

8. The exponential of a real number x can be obtained using the following infinite sum:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^n}{n!} + ...$$

(a) Write an iterative code (using loops) in MATLAB function file to calculate e^x for given x and finite terms n

(b) Write a MATLAB recursive function file for the same problem

9. Write a **MATLAB function** called **PascalTriangle** to produce Pascal's Triangle for arbitrary number of rows and to identify some of its number patterns. Your program should consist of following subfunctions.

(a) A function **myFactorial** that returns the factorial when a non-negative integer is received as input (*Hint: Consider using MATLAB's **prod** function*). The factorial of non-negative integer n is defined as follows.

$$n! = \begin{cases} 1, & \text{if } n = 0, \\ n(n-1)(n-2)\cdots 1, & \text{if } n \neq 0. \end{cases}$$

(b) A function **myBinomial** that takes two inputs, n and k, then returns the binomial coefficient (*you may use the myFactorial function written in Part (a)*). The binomial coefficient is defined by

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

(c) A function **myPascalTriangle** to display Pascal's Triangle as a lower triangle of a square matrix for an arbitrary number of rows (*you may use the myBinomial function written in Part (b)*). The nth row of the Pascal's Triangle holds the binomial coefficients

$$\binom{n}{k}, k = 0, 1, \ldots, n.$$

(d) A function **RowSumsPascal** to find the row sums of the Pascal's Triangle (*you may use the myPascalTriangle function written in Part (c)*). Identify the pattern of the sums?

(e) A function **Pascal5by5** to display the five by five matrix form Pascal's Triangle such that having first five rows and first five columns (*you may use the myPascalTriangle function written in Part (c)*). Let p_i, $i = 1, ..., 5$ be any number given by putting a row elements together (for an example the third row [1,2,1] of Pascal's Triangle can be written as 121), verify that $p_i = 11^{i-1}$ for $i = 1, 2, ..., 5$.

10. The surface area A and the volume V of a cylinder of radius r and height h are given by the formulas $A = 2\pi r(h+r)$ and $V = \pi r^2 h$ respectively. Write a MATLAB function named *CylindersAreasVolumes(vr, vh)*

that takes a vector of radii (vr) and a corresponding vector of heights (vh) of some finite number of cylinders as parameters and returns the surface areas and volumes of the cylinders using the formulas given above. Your program should consists *three different* sub functions to:

(a) calculate the area of a cylinder,

(b) calculate the volume of the cylinder, and

(c) display the results.

Test your program for $vr = [2,5,3,6,4]$ and $vh = [3,4,5,7,4]$.

11. The number e is an important mathematical constant that is the base of the natural logarithm. Write a MATLAB function *evalue(n)* to calculate approximate values of e for a given positive integer n using

    (a) the series sum

    $$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{n!} + ..., \text{ and}$$

    (b) the limit, $\lim_{x \to n}(1+1/x)^x$, where x is a real variable.

    Assuming true value of e is given by 2.7183, calculate the errors involve in both approximations for few n values.

12. Write a MATLAB function called **Cosinevalue** that gives the approximate value of the cosine of a number x using the following relation;

    $$cos(x) \approx \sum_{k=1}^{n+1} (-1)^{(k-1)} \frac{x^{2(k-1)}}{(2(k-1))!}.$$

    Your function should consist of the sub function called **myfactorial** that returns the factorial when a non-negative integer is received as an input (**Hint**: You may use MATLAB built-in function *prod*). When $n = 9$, what is the error for $x = 2\pi$?

13. (a) The midpoint rule to compute a definite integral evaluates the function at the midpoint of the integration interval:

    $$\int_a^b f(x)dx \approx (b-a) f\left(\frac{a+b}{2}\right).$$

Write a MATLAB function called **midpoint**(f,a,b) that returns the integral of f over the interval $[a,b]$ using the midpoint rule.

(b) Simpson's rule approximates a definite integral within the interval $[a,b]$ of function $f(x)$ as follows:

$$\int_a^b f(x)dx \approx \frac{b-a}{6}\left(f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right).$$

Write a MATLAB function called **Simpson**(f,a,b) that returns the integral of f over the interval $[a,b]$ using the Simpson's rule.

Test your both programs in Part (a) and Part (b) for the function $f(x)=-x^2+2x+5$ in the interval $[-2,2]$.

14. Write a user-defined MATLAB function for the following function:

$$z(x,y)=e^{x/2}cos2y+sin(x^2-3y)$$

The input of the function are x and y, the output is z.

(a) Use the function to calculate $z(-3,5)$ and $z(2,-5)$.

(b) Use the function to make a 3D plot of the function $z(x,y)$ for $-\pi \le x,y \le \pi$.